

1. 15 Venovim dijagramima prikazati sledeće skupove, a zatim odrediti koji su od njih jednaki:

$$\begin{array}{cccccc} A \setminus (B \cup C) & A \setminus B & (A \cup B) \setminus B & (A \cap B) \setminus C & \overline{(A \cap B)} & \\ (A \setminus B) \setminus C & A \cup B & (A \cup B) \setminus C & (A \setminus C) \cup (B \setminus C) & (A \setminus C) \cap (B \setminus C) & \end{array}$$

2. Na stolu se nalaze dve jednake kutije sa po 30 kupona. U prvoj se nalazi 20 kupona koji vrede 1000 dinara i 10 kupona koji vrede 2000 dinara. U drugoj kutiji se nalazi podjednak broj kupona koji vrede 1000 dinara i 2000 dinara. Petar na slučajan način bira jednu kutiju i iz nje tri izvlači 3 kupona koji predstavljaju njegov dobitak.

- a) 6 Odrediti verovatnoću da će Petar osvojiti tačno 5000 dinara.
b) 6 Ako se zna da je Petar osvojio tačno 5000 dinara, kolika je verovatnoća da je odabrao prvu kutiju?

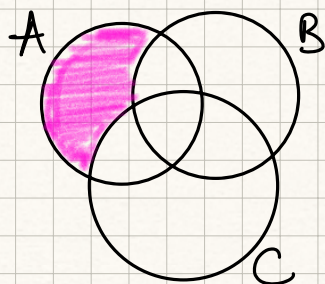
3. Kutija iz ovog zadatka sadrži 8 loptica, od kojih su 3 zelene, ostale su plave. Nikola na slučajan način izvlači jednu po jednu kuglicu dok ne izvuče zelenu. Neka X predstavlja broj izvlačenja.

- 4 a) 2 Naći raspodelu verovatnoća slučajne promenljive X , ako se loptice izvlače bez vraćanja u kutiju.
b) 7 Naći raspodelu verovatnoća slučajne promenljive X , ako se loptice izvlače sa vraćanjem u kutiju.
c) 4 Koji je naziv raspodele slučajne veličine opisane pod b) ? Odrediti njeno matematičko očekivanje $E(X)$.

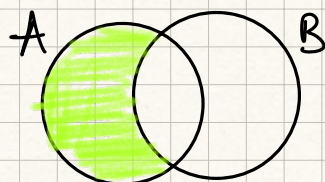
4. 10 Ako je data funkcija gustine verovatnoća neprekidne slučajne promenljive $f_X(x) = \begin{cases} 0, & x < 0 \\ 2x, & 0 \leq x < 1 \\ 0, & x \geq 1 \end{cases}$, odrediti funkciju

raspodele $F_X(x)$ i izračunati $p(-\frac{1}{2} \leq X \leq \frac{1}{2})$.

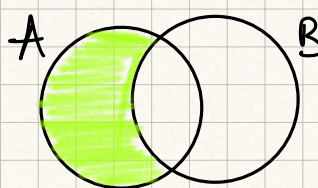
1. $A \setminus (B \cup C)$



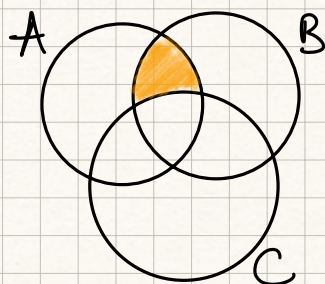
$A \setminus B$



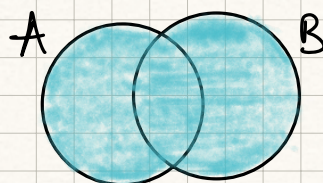
$(A \cup B) \setminus B$



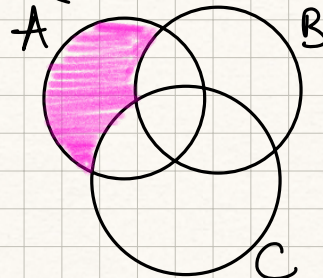
$(A \cap B) \setminus C$



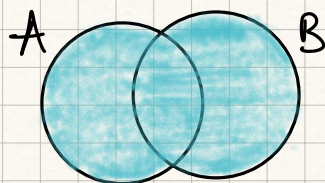
$\overline{(A \cap B)}$



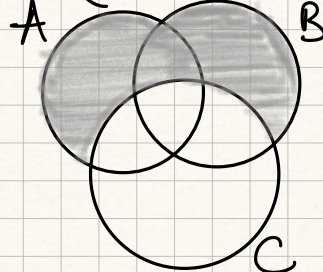
$(A \setminus B) \setminus C$



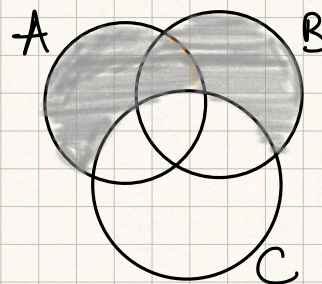
$A \cup B$



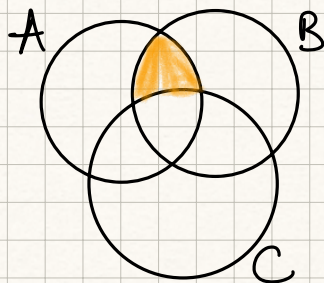
$(A \cup B) \setminus C$



$(A \setminus C) \cup (B \setminus C)$



$$(A \setminus C) \cap (B \setminus C)$$



$$A \setminus (B \cup C) = (A \setminus B) \setminus C$$

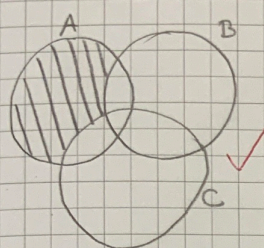
$$A \setminus B = (A \cup B) \setminus B$$

$$(A \cap B) \setminus C = (A \setminus C) \cap (B \setminus C)$$

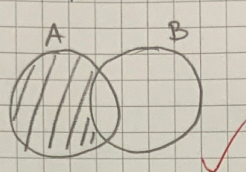
$$(\bar{A} \cap \bar{B}) = A \cup B$$

$$(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$$

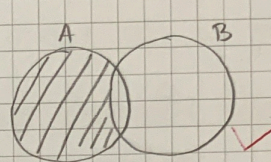
① $A \setminus (B \cup C)$



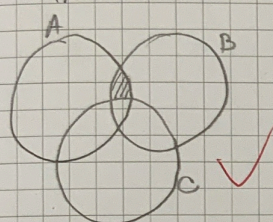
$A \setminus B$



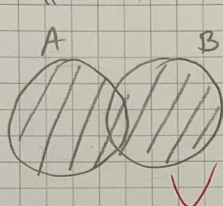
$(A \cup B) \setminus B$



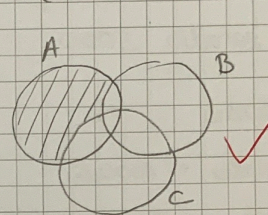
$(A \cap B) \setminus C$



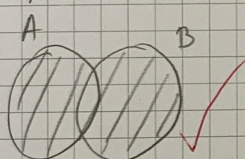
$\overline{(\bar{A} \cap \bar{B})}$



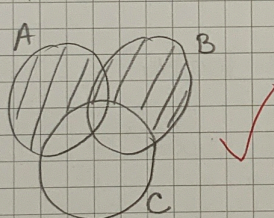
$(A \setminus B) \setminus C$



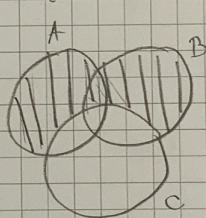
$A \cup B$



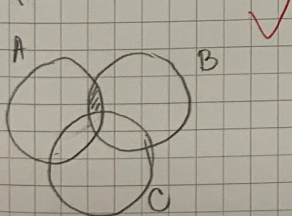
$(A \cup B) \setminus C$



$(A \setminus C) \cup (B \setminus C)$



$(A \setminus C) \cap (B \setminus C)$



15

Jednaki su: $A \setminus (B \cup C)$ i $(A \setminus B) \setminus C$ ✓

$A \setminus B$ i $(A \cup B) \setminus B$ ✓

$(A \cap B) \setminus C$ i $(A \setminus C) \cap (B \setminus C)$ ✓

$(A \cup B) \setminus C$ i $(A \setminus C) \cup (B \setminus C)$ ✓

$A \cup B$ i $\overline{(\bar{A} \cap \bar{B})}$ ✓

- (2) jednake kutije $\Rightarrow P(K_1) = 1/2$
 - po 30 kupona $P(K_2) = 1/2$

1. kutija	2. kutija
br. kupona $\Rightarrow$ vrednost [din]	
KP1: 20	1000
KP2: 10	2000

Petar $\rightarrow$ slučajno bira 1 kutiju i iz nje izvlači
 3 kupona = dobitak

- a) $A =$ "Petar osvoji tačno 5000 din" $P(A) = ?$
 b) $K_1 =$ "Petar odabere 1. kutiju"
 $P(K_1|A) = ?$

5000 = 2000 + 2000 + 1000 ✓

$$P(A) = \frac{1}{2} \cdot \frac{\binom{20}{1} \binom{10}{2}}{\binom{30}{3}} + \frac{1}{2} \cdot \frac{\binom{15}{1} \binom{15}{2}}{\binom{30}{3}}$$

$$= \frac{1}{2} \cdot \frac{20!}{19!} \cdot \frac{10!}{2!8!} + \frac{1}{2} \cdot \frac{15!}{14!} \cdot \frac{15!}{2!13!}$$

$$= \frac{1}{2} \cdot \frac{10 \cdot 20 \cdot 10 \cdot 9}{2 \cdot 28 \cdot 14} + \frac{1}{2} \cdot \frac{15 \cdot 15 \cdot 14}{2 \cdot 28 \cdot 14}$$

$$= \frac{1}{2} \cdot \frac{300}{10 \cdot 28 \cdot 14} + \frac{1}{2} \cdot \frac{15 \cdot 15 \cdot 7}{10 \cdot 28 \cdot 14} = 0,305 \checkmark$$

b) $P(K_1|A) = \frac{P(K_1 \cap A)}{P(A)} = \frac{P(K_1) \cdot P(A|K_1)}{P(A)}$

$$= \frac{\frac{1}{2} \cdot \frac{\binom{20}{1} \binom{10}{2}}{\binom{30}{3}}}{0,305} = \frac{\frac{1}{2} \cdot \frac{300}{10 \cdot 28 \cdot 14}}{0,305} = 0,363 \checkmark$$

12

U PITANJU SU K-KOMBINACIJE,
 REDOSLED NIJE BITAN!

(3) Blophira $\rightarrow$ 3 : zelelo
 5 : plave

- jedna po jedna dok ne izvuče (zelelo)
 $X =$ broj izvlačenja

a) $X: \left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \frac{21}{56} & \frac{15}{56} & \frac{10}{56} & \frac{6}{56} & \frac{3}{56} & \frac{1}{56} \end{array} \right)$
 - bez vraćanja u kutiju $\Sigma = 1$ ✓

$$P(X=1) = \frac{3}{8} = \frac{21}{56}$$

$$P(X=2) = \frac{5}{8} \cdot \frac{3}{7} = \frac{15}{56}$$

$$P(X=3) = \frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} = \frac{5}{28} = \frac{10}{56} \checkmark$$

$$P(X=4) = \frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} \cdot \frac{3}{5} = \frac{3}{28} = \frac{6}{56} \checkmark$$

$$P(X=5) = \frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} \cdot \frac{2}{5} \cdot \frac{3}{4} = \frac{3}{56} \checkmark$$

$$P(X=6) = \frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} \cdot \frac{2}{5} \cdot \frac{1}{4} \cdot 1 = \frac{1}{56} \checkmark$$

b) - vraćanje u kutiju

- Verovatnoća pri svakom izvlačenju je ista

$$P(\text{zelelo}) = \frac{3}{8} \quad P(\text{plave}) = \frac{5}{8}$$

$$P(X=1) = \frac{3}{8} \checkmark$$

$$P(X=2) = \frac{5}{8} \cdot \frac{3}{8}$$

$$P(X=3) = \left(\frac{5}{8}\right)^2 \cdot \frac{3}{8} \checkmark$$

$$P(X=4) = \left(\frac{5}{8}\right)^3 \cdot \frac{3}{8} \checkmark$$

$$P(X=5) = \left(\frac{5}{8}\right)^4 \cdot \frac{3}{8}$$

$$P(X=6) = \left(\frac{5}{8}\right)^5 \cdot \frac{3}{8}$$

$$P(X=n) = \left(\frac{5}{8}\right)^{n-1} \cdot \frac{3}{8} \checkmark$$

c) računovodnja, $E(X) = \frac{1}{p} = \frac{1}{\frac{3}{8}} = \frac{8}{3}$

4. $f_X(x) = \begin{cases} 0 & , x < 0 \\ 2x & , 0 \leq x < 1 \\ 0 & , x \geq 1 \end{cases}$

$F_X(x) = ?$

$f_X(x) \geq 0 \quad \int_{-\infty}^{\infty} f_X(x) dx = 1$

$F_X(x) = P(X \leq x) = \int_{-\infty}^x f_X(t) dt$

$\int_0^1 2x dt = 2 \int_0^1 x dt = 2 \left| \frac{x^2}{2} \right|_0^1 = \cancel{2} \cdot \frac{x^2}{\cancel{2}} = x^2$

$F_X(x) = \begin{cases} 0 & , x < 0 \\ x^2 & , 0 \leq x < 1 \\ 1 & , x \geq 1 \end{cases}$ ✓

$P(-\frac{1}{2} \leq X \leq \frac{1}{2}) = F(\frac{1}{2}) - F(-\frac{1}{2})$

$F(-\frac{1}{2}) = 0$

$P(-\frac{1}{2} \leq X \leq \frac{1}{2}) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ ✓

(10)