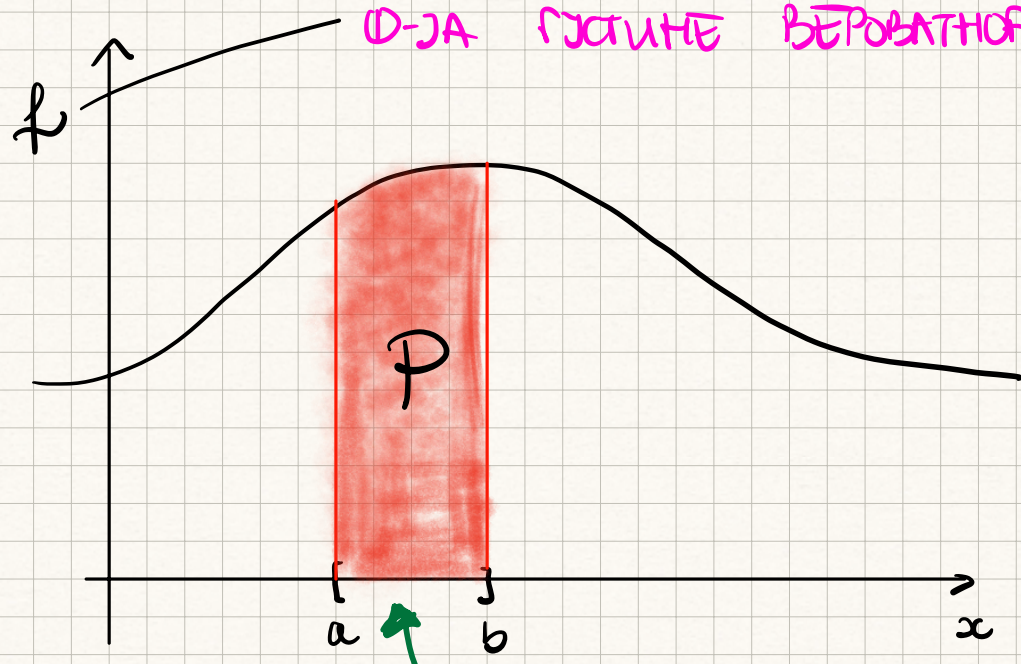


НЕПРЕКИДНА (КОНТИНУАЛНА) СЛУЧАЈНА ПРОМЕНЛИВА

X је непрек. ако је гомет. њених вредности интервал на реалној осци.

Пример: који оградних кетта у једно м расу. Колико висина је непрек. случ. величина која може да узме само коју вредност из интервала (145, 200)

PDF = "Probability density function"



Ота так интервалу о вероватноћи да се случ. променл. нађе у неком интервалу

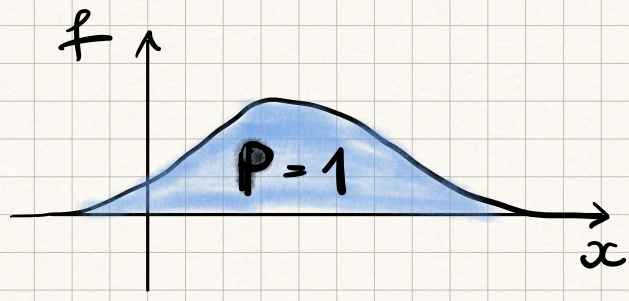
$$P(a \leq X \leq b) = \int_a^b f_X(x) dx$$

$$P(-\infty \leq X \leq \infty) = 1$$

Ипродер га дуге

ОСОБИНЕ

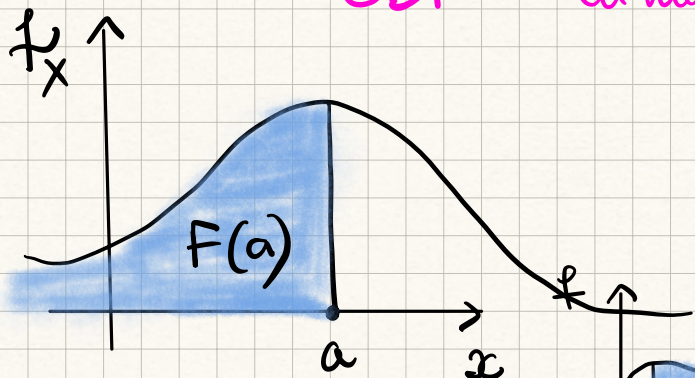
- 1° $f_X(x) \geq 0$
- 2° $\int_{-\infty}^{\infty} f_X(x) dx = 1$



1-Я РАСПРЕДЕЛЕНИЕ ПЕРЕКРУЖИТЕ СЛ. ПЕРЕМЕННЫЕ

$$F_X(x) = P(X \leq x) = \int_{-\infty}^x f_X(t) dt, \quad -\infty < x < \infty$$

CDF = "cumulative Distributions function"



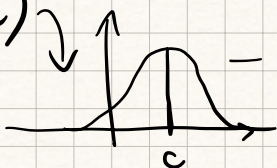
$$F(a) = \int_{-\infty}^a f_X(x) dx$$

ОСОБЕННОСТИ:

1° $P(a \leq X \leq b) = F(b) - F(a)$

2° $P(X=c) = 0, \quad \leftarrow \forall c \in \mathbb{R}$

$P(c \leq X \leq c)$

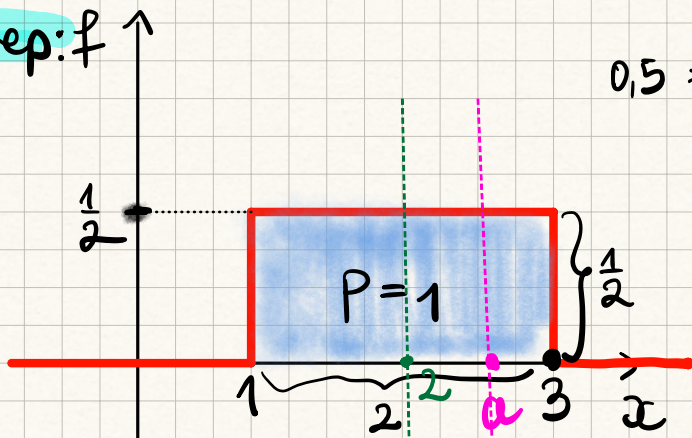


- вероятность нуля же = 0

Последствия:

3° $P(a \leq X \leq b) = P(a < X \leq b) = P(a \leq X < b) = P(a < X < b)$

Пример: f



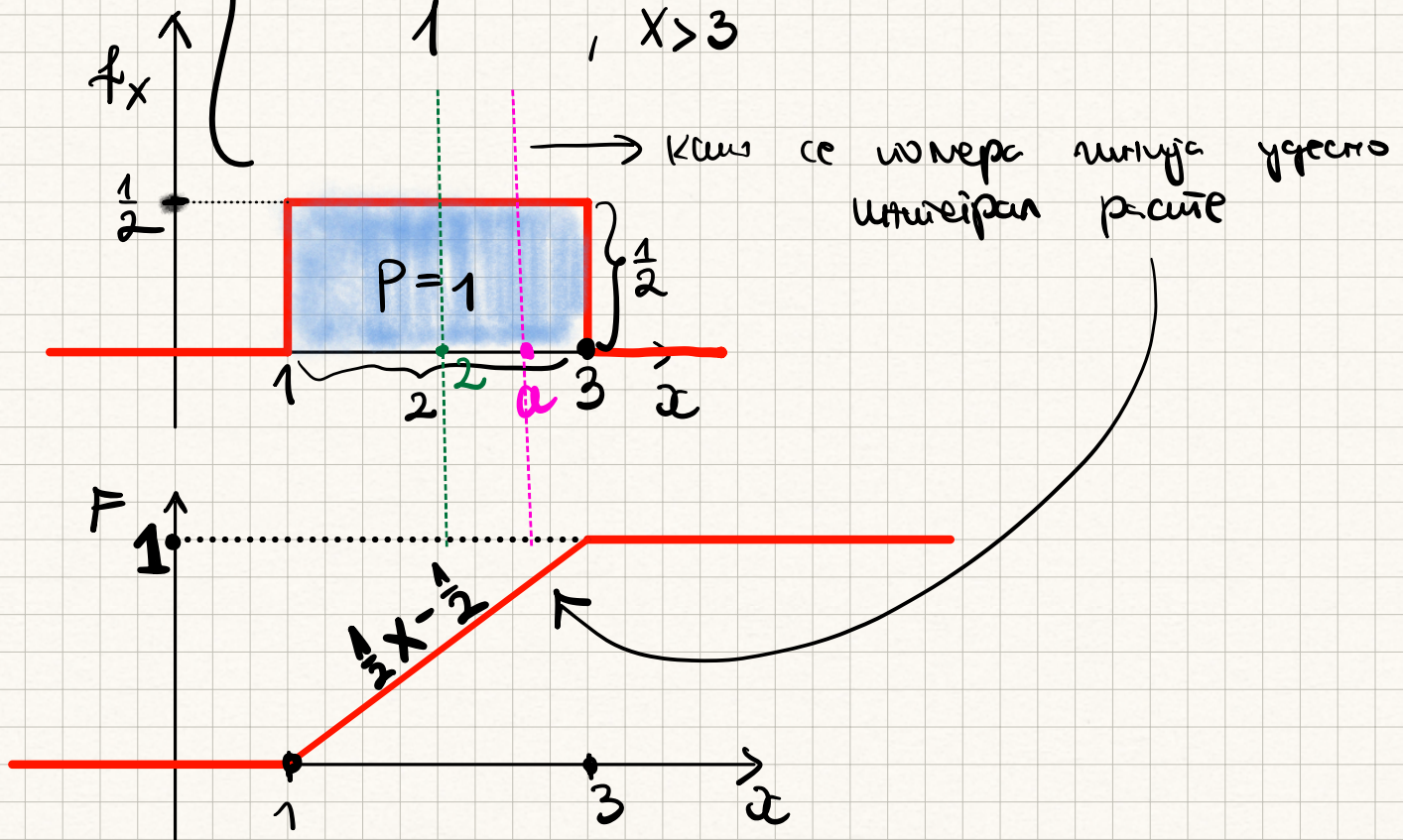
$$0.5 = P(1 \leq X \leq 2) = \int_1^2 \frac{1}{2} dx = \frac{1}{2} x \Big|_1^2 = \frac{1}{2} (2-1) = \frac{1}{2}$$

$F(a) = ? \quad a \in (1, 3)$
 $= P(-\infty \leq X \leq a)$

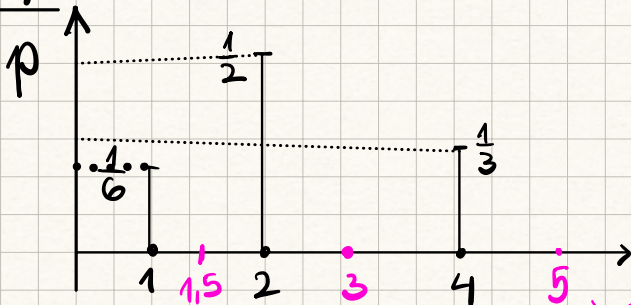
$$F(a) = \int_{-\infty}^a \frac{1}{2} dx = \int_1^a \frac{1}{2} dx = \frac{1}{2} x \Big|_1^a = \frac{1}{2}(a-1)$$

$$\Rightarrow F(x) = \frac{1}{2}(x-1) = \frac{1}{2}x - \frac{1}{2}$$

$$F_x(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{2}x - \frac{1}{2} & 1 \leq x \leq 3 \\ 1 & x > 3 \end{cases}$$



пример:



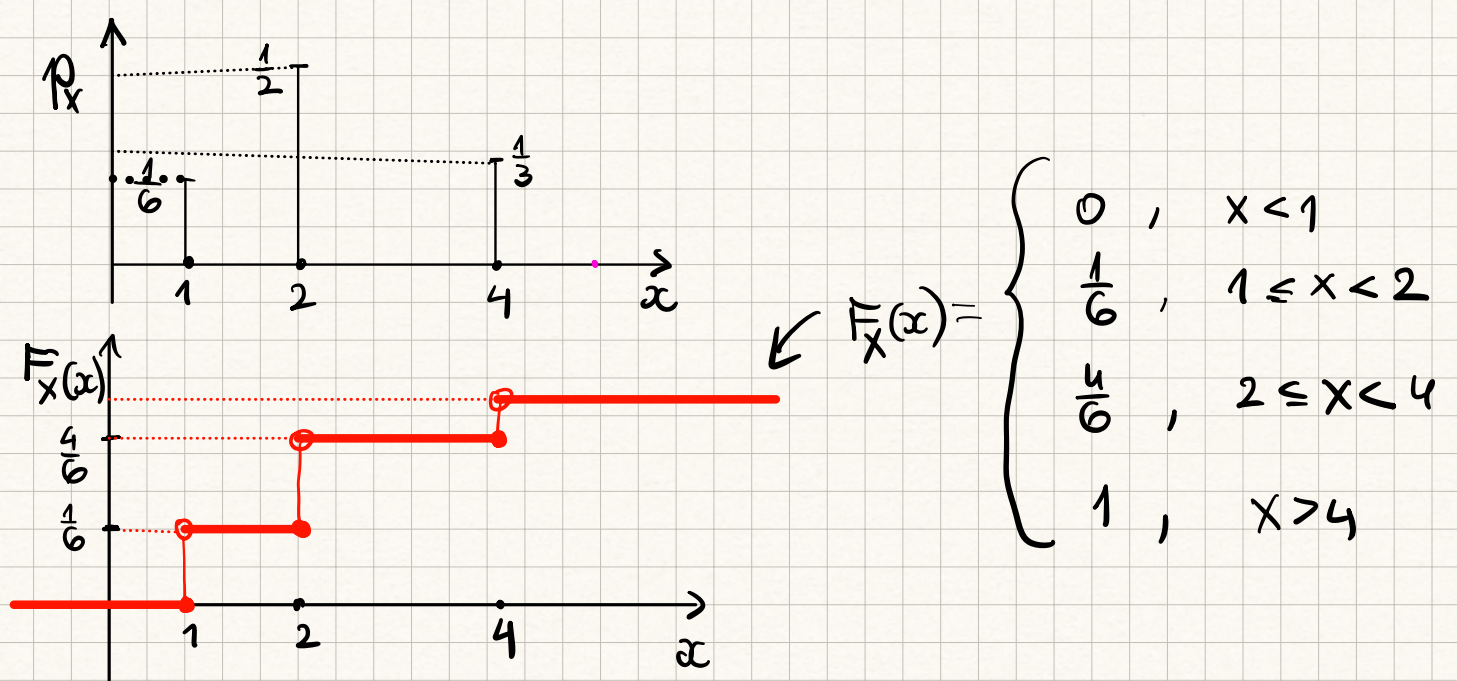
$$F_x(x) = P(X \leq x)$$

$$F(3) = P(X \leq 3) = P(X=1) + P(X=2) = \frac{1}{6} + \frac{1}{2}$$

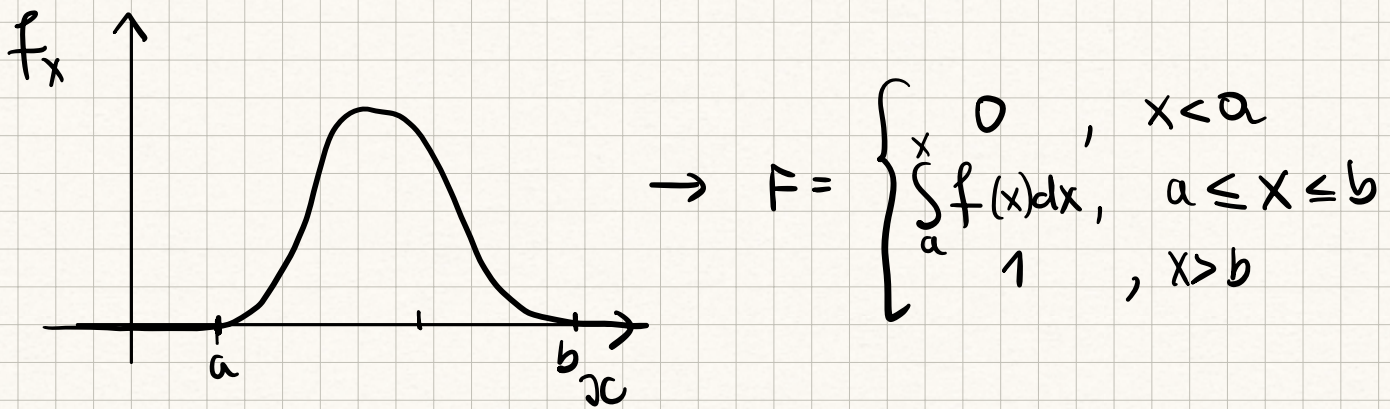
$$F(1.5) = P(X \leq 1.5) = P(X=1) = \frac{1}{6}$$

$$F(5) = P(X \leq 5) = P(X=1) + P(X=2) + P(X=4) = \frac{1}{6} + \frac{1}{2} + \frac{1}{3} = 1$$

$$F_x(x) = P(X \leq x) = \sum p_x(x)$$



у опшном случају $f + F$



МАТЕМАТИЧКО ОЧЕКИВАЊЕ + ДИСПЕРЗИЈА

ДИСКРЕТАН

$$E(x) = \sum x \cdot p_x(x)$$

$$D(x) = \sum (x - E(x))^2 \cdot p_x(x)$$

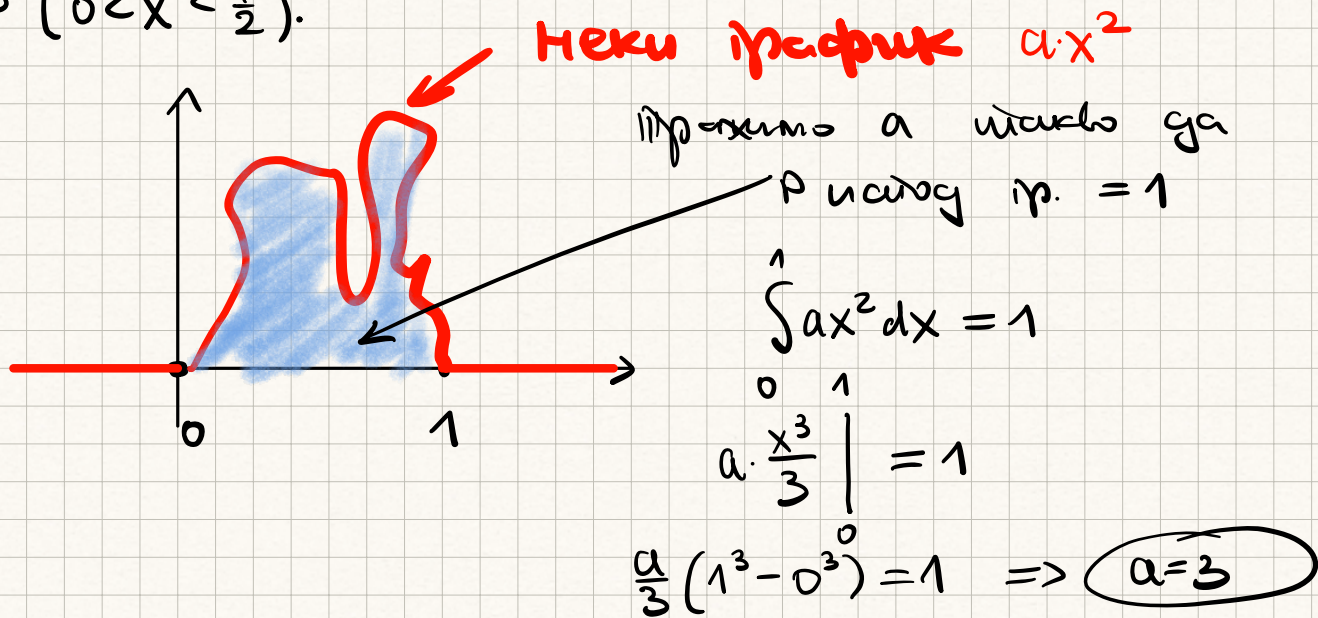
НЕПРЕКИДАН

$$\longrightarrow E(x) = \int_{-\infty}^{\infty} x \cdot f_x(x) dx$$

$$\longrightarrow D(x) = \int_{-\infty}^{\infty} (x - E(x))^2 \cdot f_x(x) dx$$

Пример: Определим а како је $f_X(x) = \begin{cases} ax^2, & 0 \leq x \leq 1 \\ 0, & \text{иначе} \end{cases}$

функција густина расподеле вероватноћа Непр. сл. променљиве.
 Замисли како функција расподеле (F) и израчунајте
 $P(0 < X < \frac{1}{2})$.



$$\Rightarrow f_X(x) = \begin{cases} 0, & x < 0 \\ 3x^2, & 0 \leq x \leq 1 \\ 0, & x > 1 \end{cases}$$

Како F?

$$F(x) = \begin{cases} 0, & x < 0 \\ x^3, & 0 \leq x \leq 1 \\ 1, & x > 1 \end{cases}$$

$\int_0^x 3t^2 dt = x^3$

Израчунајте $P(0 < X < \frac{1}{2})$?

$$P(0 < X < \frac{1}{2}) = F(\frac{1}{2}) - F(0) = \frac{1}{2}^3 - 0^3 = \frac{1}{8}$$

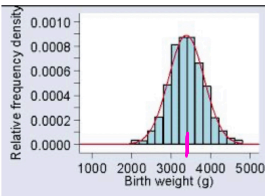
$$\text{Израчунајте } P(-1 < X < 3) = F(3) - F(-1) = 1 - 0 = 1$$

$$P(\frac{1}{2} < X < 3) = F(3) - F(\frac{1}{2}) = 1 - \frac{1}{8} = \frac{7}{8}$$

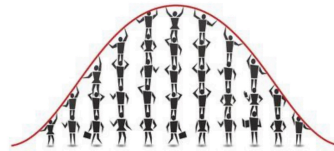
$$P(X > \frac{1}{4}) = 1 - P(X \leq \frac{1}{4}) = 1 - F(\frac{1}{4}) = 1 - (\frac{1}{4})^3 = \dots$$

← Није < ?

НОРМАЛНА (ГАУСОВА) РАСПОДЕЈА

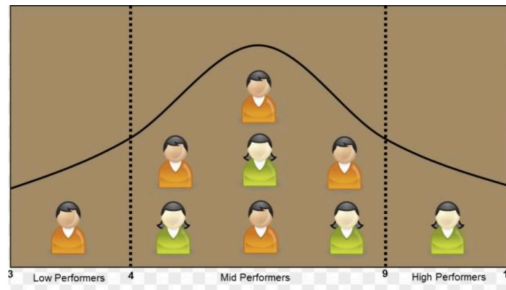


The normal birth weight of a newborn ranges from 2.5 to 3.5 kg. The majority of newborns have normal birthweight whereas only a few percent of newborns have a weight higher or lower than normal. Hence, birth weight also follows the normal distribution curve.



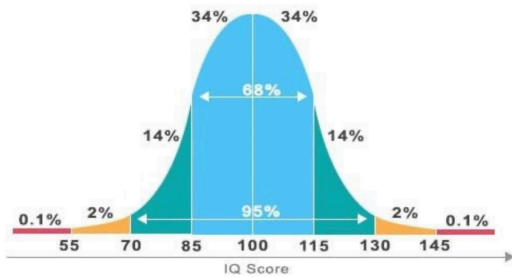
The height of people is an example of normal distribution. Most of the people in a specific population are of average height. The number of people taller and shorter than the average height people is almost equal, and a very small number of people are either extremely tall or extremely short. Several genetic and environmental factors influence height. Therefore, it follows the normal distribution.

9. Student's Average Report

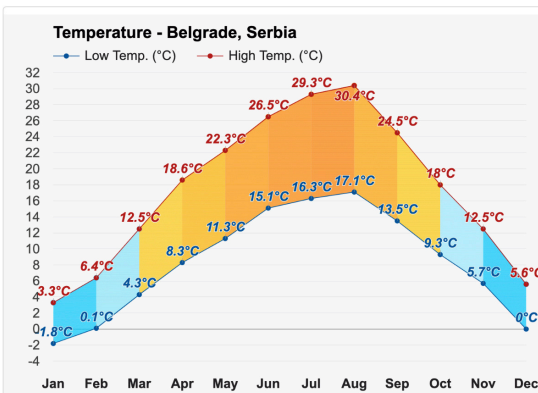


4. IQ

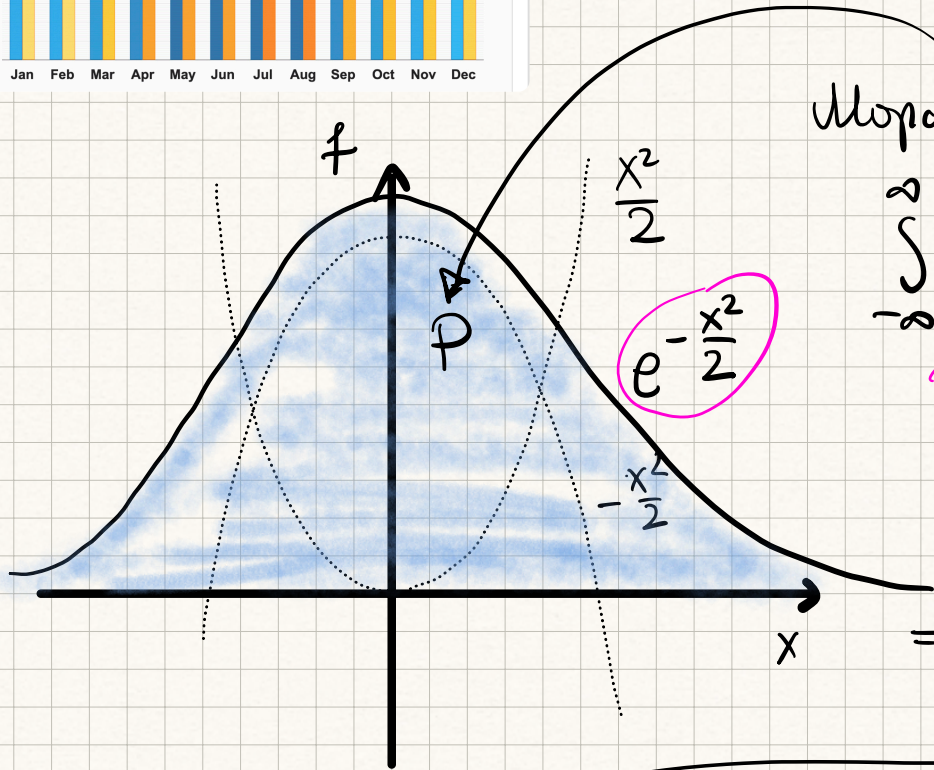
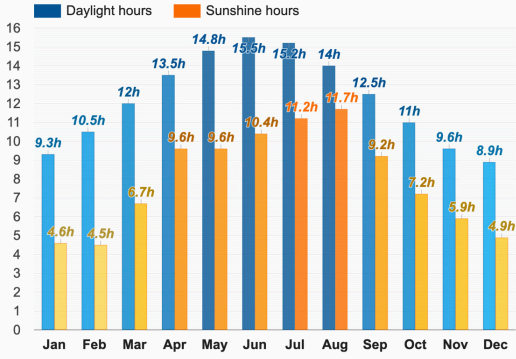
IQ GRAPH



Prosečna temperatura Beograd, Srbija



Daylight hours / Sunshine hours - Belgrade, Serbia



Улога га дуге $\rho=1$

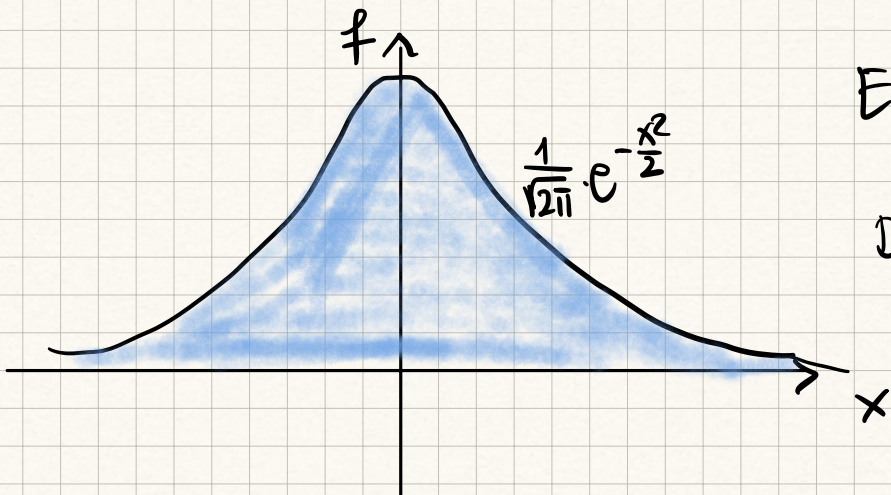
$$\int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} dx = \sqrt{2\pi}$$

уместо $e^{-\frac{x^2}{2}}$ користимо $\frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}}$

$$\Rightarrow \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}} = 1$$

$$f_x(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}} - \text{ФЈА ФУНКЦИЈЕ}$$

Сл. величина која има облик ф-је гране има СТАНДАРТУ НОРМАЛМУ РАСПОДЕЛУ!



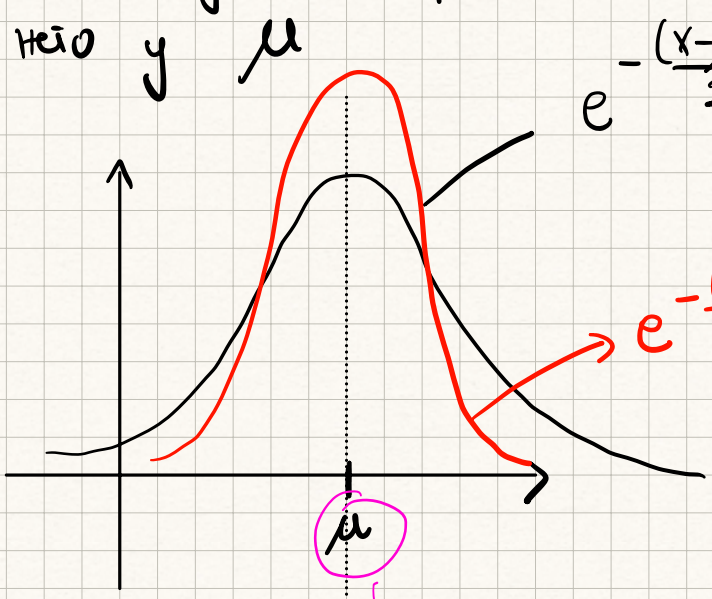
$E(x) = 0$ Говорно је средња вредност

$D(x) = ?$
 $= \int (x - E(x))^2 \cdot \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}} dx$
 \dots
 $= 1$

$X \sim \mathcal{N}(0, 1)$ → дисперзія

↓
 математичне очікування
 X має СТАНДАРДНУ НОРМАЛНУ РАСПОДЕЛУ
 ↓
 значи $E(X)=0, D(X)=1$

- Хелімо у попериме графік, га $E(X)$ не буде у 0 нею у μ



$e^{-\frac{(x-\mu)^2}{2}}$

Щита аю хелімо у сушлю/спрашимо?

$e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

↓
 Га ми обернута консу графік для $=1$

$\frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

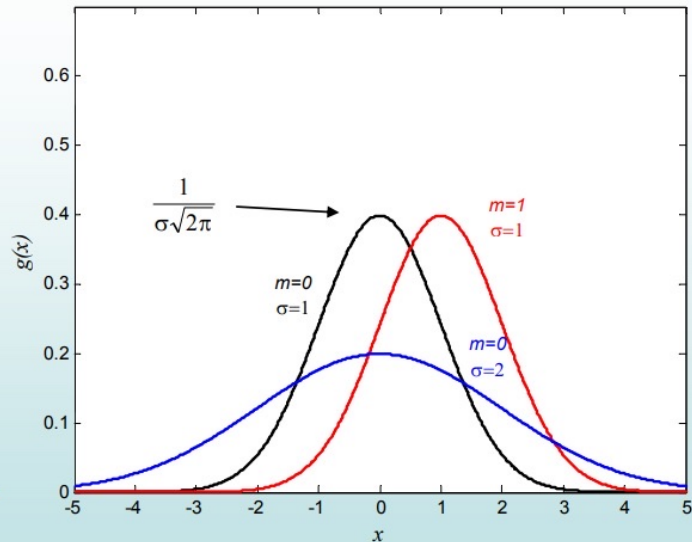
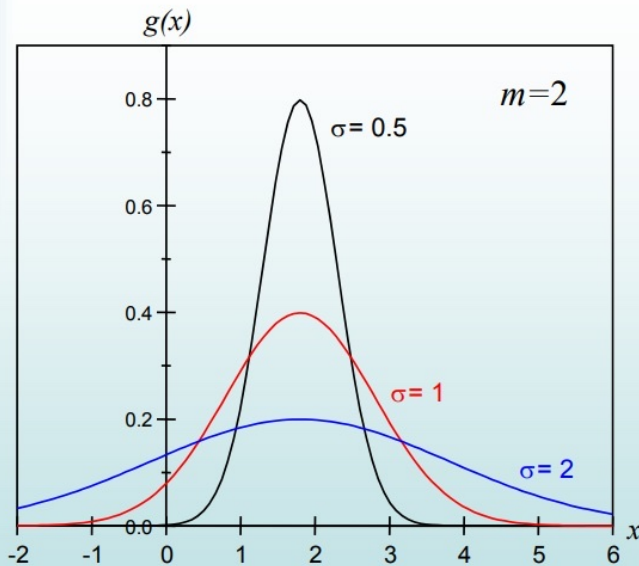
Зодиме аю консу ф-ю писати:

$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

Сл. вел. кон аю обиклу ф-ю писати има НОРМАЛНУ РАСПОДЕЛУ

$X \sim \mathcal{N}(\mu, \sigma^2)$
 ↑ матем. очікування ↙ дисперзія

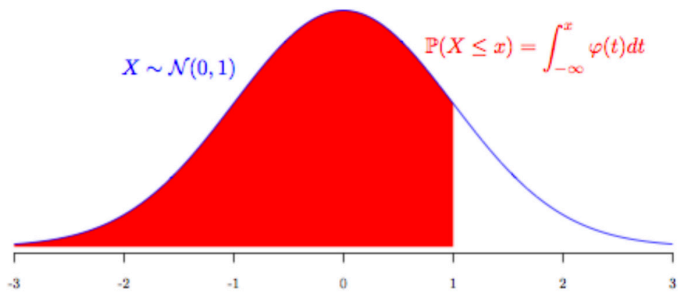
Normalna raspodela za različite vrednosti parametara m i σ^2



Φ ja poznajemo sa $X \sim \mathcal{N}(0,1)$ uzimega treba:

$$F_X(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$$

$$P(X \in [1,2]) , X \sim \mathcal{N}(0,1)$$



$F(0,41) = P(X \leq 0,41)$

0,41 иррационално
у брским

0,01 иррационално у
коноту

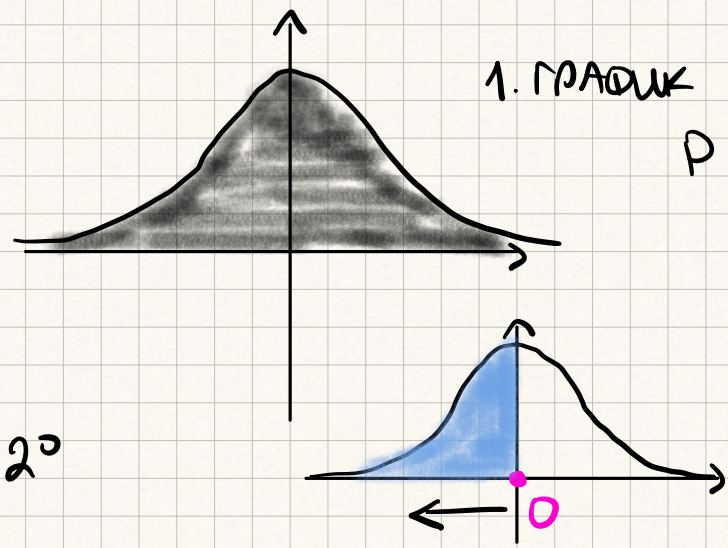
$P(X \leq 0,41) = 0,6591$

$P(0) = \frac{1}{2}$

$P(3,09) = 0,9999$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

ОСОБИНЕ СТАНДАРДНЕ НОРМАЛНЕ РАСПОДЕЛЕ:

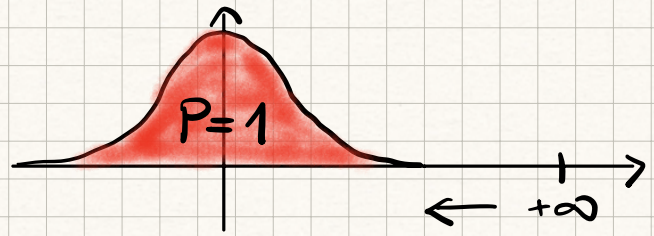


1. ПРАДОК СИМЕТАЧАН ОКО X-ОСЕ
P и целоу просршиа = 1

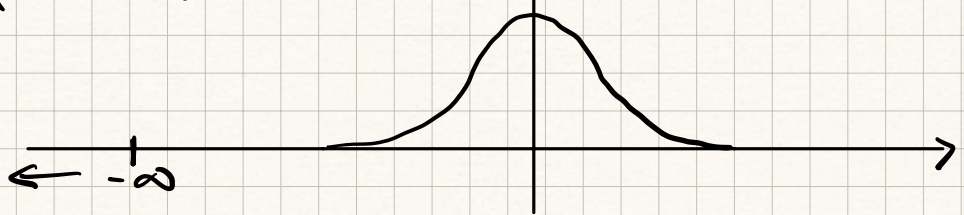
$P(0) = \frac{1}{2} = P(X \leq 0)$

20

3° $F(+\infty) = P(X < +\infty) = 1$

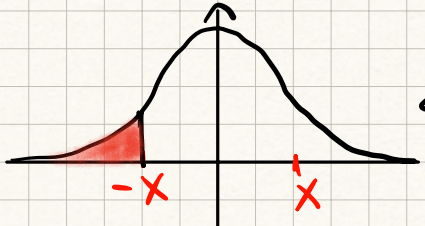


$F(-\infty) = P(X < -\infty) = 0$

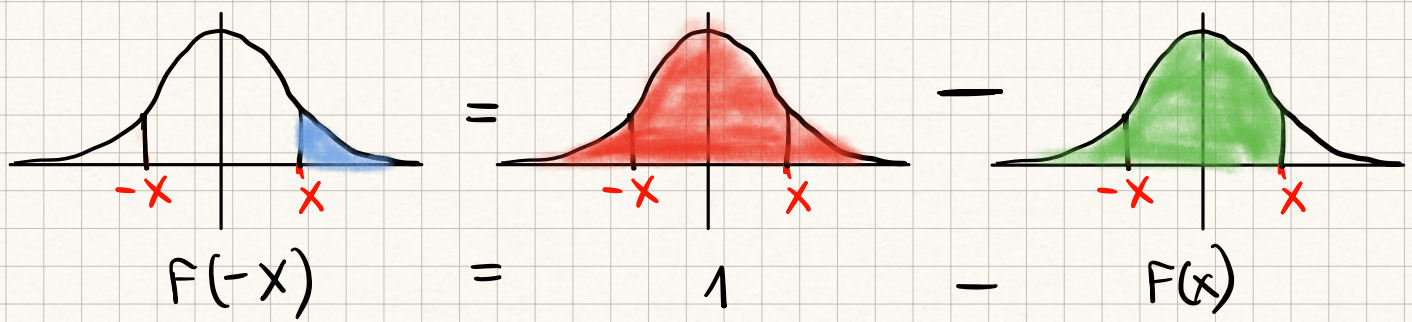
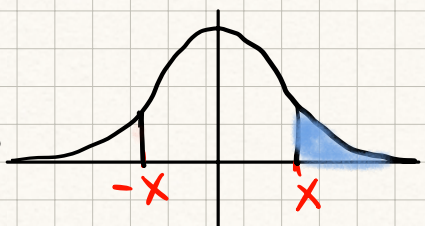


4° $F(-x) = ?$

$F(-x) = P(X \leq -x)$

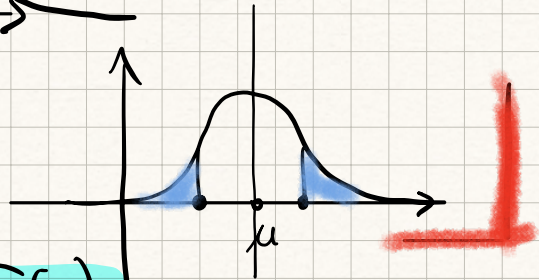
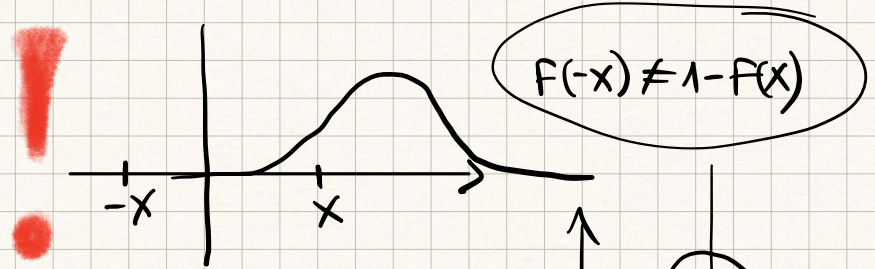


je учиво/једнако



$F(-x) = 1 - F(x)$

F ово обично важи само за сим. норм. расподелу.



5° $P(a < X < b) = F(b) - F(a)$

