PRIBLIŽNO REŠAVANJE NELINEARNIH J-NA:

$f(x)=0$

x^* - TAČNO REŠENJE / [illegible]

$f(x^*)=0$

T: O EGZISTENCIJI REŠENJA

NEKA JE DATA $f: [a,b] \to \mathbb{R}$ NEPREKIDNA I NEKA JE $f(a)\cdot f(b) < 0$

TADA J-NA $f(x)=0$ IMA BAR JEDNO REŠENJE NA INTERVALU $[a,b]$

OCENA GREŠKE:

LAGRANŽOVA TEOREMA: $\dfrac{f(x_p) - \cancel{f(x^*)}^{\to 0}}{x_p - x^*} = f'(z) \Rightarrow$

$$|f(x_p)| = |f'(z)|\cdot|x_p - x^*| \geq m_1 |x_p - x^*|$$

$$m_1 = \min_{x\in[a,b]} |f'(x)|$$

DAKLE,

$$\boxed{|x_p - x^*| \leq \frac{f(x_p)}{m_1}}$$

LJENJA INTERVALA:

$$[a,b] \longrightarrow [a_0,b_0]$$

$$f(x)=0$$

$$[a_1,b_1] \longrightarrow \left[a, \frac{a+b}{2}\right]$$

$$\vdots$$

$$[a_n,b_n]$$

... $f:[a,b]\to\mathbb{R}$ NEPREKIDNA, NEKA JE $f(a)\,f(b)<0$ I NEKA IZVOD NE MENJA ZNAK NA INTERVALU $[a,b]$. TADA NIZ INTERVALA ...] KONVERGIRA KA TAČNOM REŠENJU $\bar{x}$ JNE $f(x)=0$

$$f(a_n)\cdot f(b_n)<0 \qquad (*)$$

$$b_n-a_n=\frac{b-a}{2^n} \Rightarrow \lim_{n\to+\infty} a_n = \lim_{n\to+\infty} b_n = \bar{x}$$

KAD $n\to+\infty$ U $(*)$

$$f(\bar{x})\cdot f(\bar{x}) \leq 0 \Rightarrow f(\bar{x})=0 \Rightarrow \bar{x}=x^*$$

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METODOM POLOVLJENJA INTERVALA REŠITI J-NU $x^3+2x^2-x-1=0$ NA INT

n	a_n	b_n	$\frac{a_n+b_n}{2}$
0	0	1	0,5
1	0,5	1	0,75
2	0,75	1	0,875
3	0,75	0,875	0,8125
4	0,75	0,8125	0,78125
5	0,78125	0,8125	0,796875
6	0,796875	0,8125	0,80
7	0,79	0,80	

$\varepsilon = 0{,}5 \cdot 10^{-2}$

$0{,}80 - 0{,}79 = 0{,}01 < \varepsilon$ ✓ KRAJ KAD JE VELIČINA INTERVALA MANJA OD ε

$0{,}5^3 + 2 \cdot 0{,}5^2 - 0{,}5 - 1 = -0{,}875 < 0$ (MENJAMO a_n, b_n OSTAJE)

$0{,}75^3 + 2 \cdot 0{,}75^2 - 0{,}75 - 1 = -0{,}203 < 0$

$0{,}875^3 + 2 \cdot 0{,}875^2 - 0{,}875 - 1 = 0{,}326 > 0$

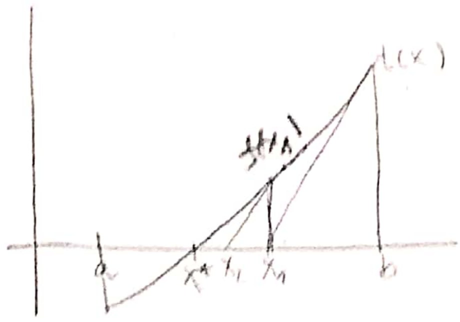
$0{,}8125^3 + 2 \cdot 0{,}8125^2 - 0{,}8125 - 1 = 0{,}044 > 0$

$0{,}78125^3 + 2 \cdot 0{,}78125^2 - 0{,}78125 - 1 = -0{,}0837 < 0$

$0{,}796875^3 + 2 \cdot 0{,}796875^2 - 0{,}796875 - 1 = -0{,}0208 < 0$

$0{,}80^3 + 2 \cdot 0{,}80^2 - 0{,}80 - 1 = -0{,}008 < 0$

Njutnova metoda tangente:



$b = x_0$

$$t(x) = f(x_0) + f'(x_0)(x - x_0)$$

$$t(x) = 0: f(x_0) + f'(x_0)(x - x_0) = 0$$

$$f'(x_0)(x - x_0) = -f(x_0) \quad | : f'(x_0)$$

$$\boxed{x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}}$$

$\vdots$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, \dots$$

T: Neka važe sledeći uslovi:

1° $f: [a,b] \to \mathbb{R}$ je neprekidna i važi $f(a) \cdot f(b) < 0$

2° $f'(x)$ je neprekidna fja na intervalu $[a,b]$

3° $f''(x)$ postoji na intervalu $[a,b]$

4° $f'(x)$ i $f''(x)$ ne menjaju znak na intervalu $[a,b]$

5° $f(x_0) \cdot f''(x_0) > 0$

Tada niz x_n konvergira ka tačnom rešenju j-ne $f(x) = 0$.

ETODOM TANGENTE NAĆI REALNO REŠENJE J-NE $X^3-4X-4=0$ SA GR

$\varepsilon=10^{-4}$ NA INTERVALU $[2\,;2{,}5]$.

$\varepsilon=10^{-4}=0{,}0001$

$f(x)=x^3-4x-4$

$f'(x)=3x^2-4$, LEVI KRAJ JE min $\Rightarrow m_1=3\cdot 2^2-4=3\cdot 4-4=8$

$f''(x)=6x$

$x_0=2{,}5$ (KADA JE $x_0=2$ $f(x_0)\,f''(x_0)<0$ ✗)

$$x_1=x_0-\frac{f(x_0)}{f'(x_0)}$$

$f(x_0)=2{,}5^3-4\cdot 2{,}5-4=1{,}625$

$f'(x_0)=3\cdot 2{,}5^2-4=14{,}75$

$$x_1=2{,}5-\frac{1{,}625}{14{,}75}=2{,}389831$$

$$x_2=x_1-\frac{f(x_1)}{f'(x_1)}$$

$\frac{f(x_1)}{m_1}>\varepsilon$ ZA [illegible]

$f(x_1)=2{,}389831^3-4\cdot 2{,}389831-4=0{,}089699$

$f'(x_1)=3\cdot 2{,}389831^2-4=13{,}1338766$

$$x_2=2{,}388831-\frac{0{,}089699}{13{,}1338766}=2{,}383001$$

$f(x_2)=2{,}383001^3-4\cdot 2{,}383001-4=0{,}0003289$

$$\frac{f(x_2)}{m_1}=\frac{0{,}0003289}{8}=0{,}0000412<\varepsilon \checkmark$$

METODA SEČICE:

$$X_{n+1} = X_n - \frac{f(X_n)}{f'(X_n)}, \quad n = 0, 1, \ldots$$ – NJUTNOVA METODA

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f'(x_1) = \lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1} \Rightarrow f'(x_1) \approx \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$\boxed{f'(x_n) \approx \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}}$$ ILI $$\boxed{f'(x_n) \approx \frac{f(x_n) - f(\bar{x})}{x_n - \bar{x}}}$$

$$X_{n+1} = X_n - \frac{f(X_n)}{\frac{f(X_n) - f(X_{n-1})}{X_n - X_{n-1}}}$$

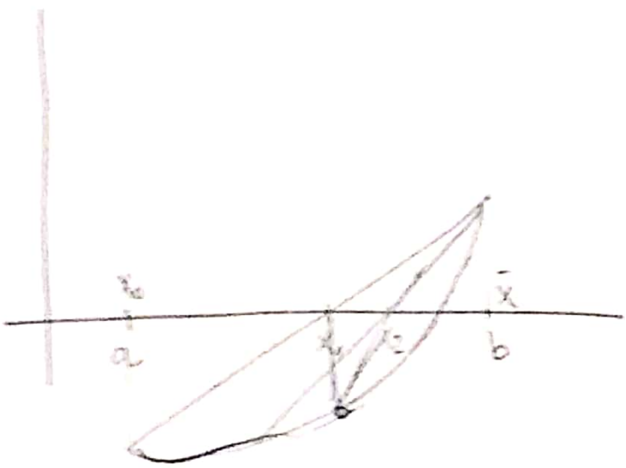
$$\boxed{X_{n+1} = X_n - f(X_n) \frac{X_n - X_{n-1}}{f(X_n) - f(X_{n-1})}}$$

METODA REGULA FALSI :

x_{n-1} - FIKSIRANA TAČKA $(\bar{x})$ - ODNOSNO JEDAN OD KRAJEVA INTERVALA [a

$$\boxed{x_{n+1} = x_n - f(x_n)\frac{x_n - \bar{x}}{f(x_n) - f(\bar{x})}}$$

AKO SU ZADOVOLJENI SVI USLOVI TEOREME ZA NJUTNOVU METODU, PRI ČEMU TAČKA $\bar{x}$ ZADOVOLJAVA USLOV $f(\bar{x}) \cdot f''(\bar{x}) > 0$ I x_0 JE DRUGI KRAJ INTERVAL OVA METODA ĆE KONVERGIRATI KA TAČNOM REŠENJU.



METODOM REGULA FALSI NAĆI PRIBLIŽNO REŠENJE JNE $x^5 - x + 1$ TAČNOŠĆU $\varepsilon = 10^{-4}$ NA INTERVALU $[-1,2; -1]$.

m_1 - minimum

$f(x) = x^5 - x + 1 = 0$

$f'(x) = 5x^4 - 1$ ⇒ spušta se, pa je min -1

$f''(x) = 20x^3$

$m_1 = 5(-1)^4 - 1 = 5 - 1$

$m_1 = 4$

$\bar{x} = -1,2 \Rightarrow x_0 = -1$

$\left(\bar{x} = -1 \quad \cancel{} \quad f(\bar{x}) \cdot f''(\bar{x}) < 0 \right)$

$f(\bar{x}) \cdot f''(\bar{x}) > 0$

$f(-1,2) = (-1,2)^5 - (-1,2) + 1 = -2,488 + 1,2 + 1 = -0,288$

$f''(-1,2) = 20(-1,2^3) = -34,56$

$f(-1,2) \cdot f''(-1,2) = -0,288 \cdot (-34,56) = 9,95 > 0$

$$x_1 = x_0 - f(x_0) \frac{x_0 - a}{f(x_0) - f(a)} = -1 - 1 \cdot \frac{-1 - (-1,2)}{1 - (-0,288)} = -1 - \frac{0,2}{1,288} = -1,155$$

$f(x_0) = -1 + 1 + 1 = 1$

$f(x_1) = (-1,155)^5 - (-1,155) + 1 = 0,0995$

$$\frac{f(x_1)}{4} = \frac{0,0995}{4} = 0,024875 > \varepsilon$$

$$x_2 = x_1 - f(x_1) \frac{x_1 - a}{f(x_1) - f(a)} = -1,155 - 0,0995 \frac{-1,155 + 1,2}{0,0995 + 0,288} = -1,1666$$

$f(x_2) = (-1,1666)^5 + 1,1666 + 1 = 0,00582$

$$\frac{f(x_2)}{4} = \frac{0,00582}{4} = 0,001455 > \varepsilon$$

$$x_3 = x_2 - f(x_2) \frac{x_2 - a}{f(x_2) - f(a)} = -1,1666 - 0,00582 \frac{-1,1666 + 1,2}{0,00582 + 0,288} = -1,1672$$

$f(x_3) = (-1,16726)^5 + 1,16726 + 1 = 0,000364$

$$\frac{f(x_3)}{4} = \frac{0,000364}{4} = 0,000091 > \varepsilon$$

$$X_4 = X_3 - f(X_3)\,\frac{X_3 - a}{f(X_3) - f(a)} = -1{,}16726 - 0{,}000364\,\frac{-1{,}16726 + 1{,}2}{0{,}000364 + 0{,}25}$$

$$f(X_4) = (-1{,}1673)^5 + 1{,}1673 + 1 = 0{,}000033$$

$$\frac{f(X_4)}{4} = \frac{0{,}000033}{4} = 0{,}00000825 < \varepsilon \checkmark$$

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