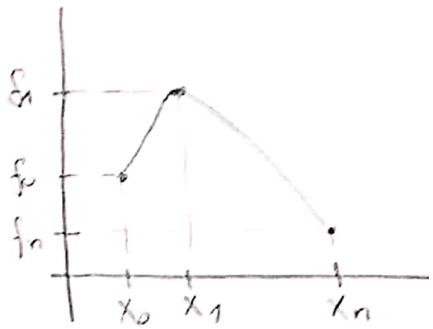
LAGRANŽOV INTERPOLACIONI POLINOM:

$x_0, x_1, \dots, x_n$ - ČVOROVI INTERPOLACIJE

$f_0, f_1, \dots f_n$ - VREDNOSTI FJE U ČVOROVIMA



- NEKA SU POLINOMI $L_k(x)$ POLINOMI n-TOG STEPENA ZA KOJE VAŽI DA $L_k(x_i) = 1$ KAD JE $k = i$; $L_k(x_i) = 0$ KAD JE $k \neq i$.

- POLINOM $L_0(x)$ JE POLINOM STEPENA n, KOJI ISPUNJAVA SLEDEĆI USLOV

$$L_0(x_0) = 1 \;,\; L_0(x_1) = L_0(x_2) = \dots = L_0(x_n) = 0$$

- ZA POLINOM $L_1(x)$ n-TOG REDA VAŽI:

$$L_1(x_0) = 0 \;,\; L_1(x_1) = 1 \;,\; L_1(x_2) = \dots = L_1(x_n) = 0$$

$$L_0(x) : C_0 x^n + \dots$$

$$L_1(x) : C_1 x^n + \dots$$

$$L_k(x) : C_k (x - x_0) \cdot \dots \cdot (x - x_{k-1}) \cdot (x - x_{k+1}) \cdot \dots \cdot (x - x_n)$$

$$L_0(x) = C_0 (x - x_1) \cdot \dots \cdot (x - x_n)$$

$$1 = L_k(x_k) = C_k (x_k - x_0) \cdot \dots \cdot (x_k - x_{k-1}) \cdot (x_k - x_{k+1}) \cdot \dots \cdot (x_k - x_n)$$

$$C_k = \frac{1}{(x_k - x_0) \cdot \dots \cdot (x_k - x_{k-1})\,(x_k - x_{k+1}) \cdot \dots \cdot (x_k - x_n)}$$

$$L_k(x) = \frac{(x - x_0) \cdot \dots \cdot (x - x_{k-1}) \cdot (x - x_{k+1}) \cdot \dots \cdot (x - x_n)}{(x_k - x_0) \cdot \dots \cdot (x_k - x_{k-1}) \cdot (x_k - x_{k+1}) \cdot \dots \cdot (x_k - x_n)}$$

$k = 0, 1, \dots, n$

$$\boxed{L(x) = f_0 \cdot L_0(x) + \dots + f_n L_n(x)}$$

Odrediti Lagranžov oblik interpolacionog polinoma za čvorove interpolacij date u tabeli:

	x_0	x_1	x_2
x	0	1	3
$f(x)$	4	3	−2
	f_0	f_1	f_2

$$P_2(x) = f(x_0)\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} + f(x_1)\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} + f(x_2)\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

$$P_2(x) = 4\frac{(x-1)(x-3)}{(0-1)(0-3)} + 3\frac{(x-0)(x-3)}{(1-0)(1-3)} - 2\frac{(x-0)(x-1)}{(3-0)(3-1)}$$

$$P_2(x) = 4\cdot\frac{x^2-4x+3}{3} + 3\frac{x^2-3x}{-2} - 2\frac{x^2-x}{6}$$

$$P_2(x) = \frac{4}{3}x^2 - \frac{16}{3}x + 4 - \frac{3}{2}x^2 + \frac{9}{2}x - \frac{1}{3}x^2 + \frac{1}{3}x$$

$$P_2(x) = -\frac{1}{2}x^2 - \frac{1}{2}x + 4$$

Aproksimirati fju $f(x) = \frac{4x}{x+1}$ Lagrangeovim polinomom u čvorovima

0, 1, 3 i naći vrednost fje $f(2)$.

	x_0	x_1	x_2
x	0	1	3
$f(x)$	0	2	3
	f_0	f_1	f_2

$\frac{4 \cdot 0}{0+1} = 0 \qquad \frac{4 \cdot 1}{1+1} = 2 \qquad \frac{4 \cdot 3}{3+1} = 3$

$$P_2(x) = f(x_0)\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} + f(x_1)\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x-x_2)} + f(x_2)\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

$$P_2(x) = 0\cdot\frac{(x-1)(x-3)}{(0-1)(0-3)} + 2\frac{(x-0)(x-3)}{(1-0)(1-3)} + 3\cdot\frac{(x-0)(x-1)}{(3-0)(3-1)}$$

$$P_2(x) = 0 + 2\cdot\frac{x^2-3x}{-2} + 3\cdot\frac{x^2-x}{6}$$

$$P_2(x) = -x^2 + 3x + 0{,}5x^2 - 0{,}5x$$

$$P_2(x) = -0{,}5x^2 + 2{,}5x \quad \Longleftarrow \text{ u svesci } -\frac{x^2}{6} + \frac{9x}{6}$$

$$f(2) = \frac{4\cdot 2}{2+1} = \frac{8}{3} = 2{,}67$$

Provera: $P_2(2) = -0{,}5\cdot 2^2 + 2{,}5\cdot 2$

$P_2(2) = 3$

NJUTNOV INTERPOLACIONI POLINOM ZA NEJEDNAKE RAZMAKE:

$f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$ - PODELJENA RAZLIKA DRUGOG REDA (UKLJUČUJE DVA ČVORA)

$f[x_1, x_2] = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$ - PODELJENA RAZLIKA TREĆEG REDA

$f[x_0, x_1, \dots, x_n] = \frac{f[x_1, x_2, \dots, x_n] - f[x_0, \dots, x_{n-1}]}{x_n - x_0}$

$$P_n(x) = f[x_0] + (x - x_0) f[x_0, x_1] + (x - x_0)(x - x_1) f[x_0, x_1, x_2] + \dots +$$
$$+ (x - x_0) \cdot \dots \cdot (x - x_{n-1}) f[x_0, \dots, x_n]$$

F-JA JE DATA SLEDEĆOM TABELOM :

	x_0	x_1	x_2	x_3	x_4
x	0	1	2	3	5
$f(x)$	0,31	0,47	0,58	0,49	0,36

KORIŠĆENJEM NJUTNOVOG INTERPOLACIONOG POLINOMA STEPENA TRI NAĆI VREDNOST FJE $f(0,5)$.

x	$f[\,]$	$f[\,,\,]$	$f[\,,\,,\,]$	$f[\,,\,,\,,\,]$	$f[\,,\,,\,,\,,\,]$
=> 0	0,31	0,16			
1	0,47		-0,025		
		0,11		-0,025	
2	0,58		-0,1		0,0104416
		-0,09		0,02708	
3	0,49		0,00833		
		-0,065			
5	0,36				

$\frac{0,47-0,31}{1-0} = 0,16$

$\frac{0,58-0,47}{2-1} = 0,11$

$\frac{0,49-0,58}{3-2} = -0,09$

$\frac{0,36-0,49}{5-3} = -0,065$

$\frac{0,11-0,16}{2-0} = -0,025$

$\frac{-0,09-0,11}{3-1} = -0,1$

$\frac{-0,065-(-0,09)}{5-2} = 0,00833$

$\frac{-0,1-(-0,025)}{3-0} = -0,025$

$\frac{0,00833-(-0,1)}{5-1} = 0,02$

$\frac{0,02708-(-0,025)}{5-0} = 0,0$

$$P_3(X) = f[X_0] + (X-X_0) f[X_0, X_1] + (X-X_0)(X-X_1) f[X_0, X_1, X_2] + (X-X_0)(X-X_1)(X-X_2) f[X_0, X_1, X_2, X_3]$$

$$P_3(X) = 0,31 + (X-0) \cdot 0,16 + (X-0)(X-1)(-0,025) + (X-0)(X-1)(X-2)($$

$$P_3(X) = 0,31 + 0,16X - 0,025X^2 + 0,025X - 0,025X(X^2 - 3X + 2)$$

$$P_3(X) = 0,31 + 0,185X - 0,025X^2 - 0,025X^3 + 0,075X^2 - 0,05X$$

$$P_3(X) = 0,31 + 0,135X + 0,055X^2 - 0,025X^3$$

$$P_3(0,5) = 0,31 + 0,135 \cdot 0,5 + 0,055 \cdot 0,5^2 - 0,025 \cdot 0,5^3$$

$$P_3(0,5) = 0,31 + 0,0675 + 0,01375 - 0,003125$$

$$\boxed{P_3(0,5) = 0,3881}$$