Solution to Problem 5.10. (a) Let $S_n = X_1 + \cdots + X_n$ be the total number of gadgets produced in n days. Note that the mean, variance, and standard deviation of S_n is $5n$, $9n$, and $3\sqrt{n}$, respectively. Thus,

$$\begin{aligned}
\mathbf{P}(S_{100} < 440) &= \mathbf{P}(S_{100} \leq 439.5) \\
&= \mathbf{P}\left(\frac{S_{100} - 500}{30} < \frac{439.5 - 500}{30}\right) \\
&\approx \Phi(\frac{439.5 - 500}{30}) \\
&= \Phi(-2.02) \\
&= 1 - \Phi(2.02) \\
&= 1 - 0.9783 \\
&= 0.0217.
\end{aligned}$$

(b) The requirement $\mathbf{P}(S_n \geq 200 + 5n) \leq 0.05$ translates to

$$\mathbf{P}\left(\frac{S_n - 5n}{3\sqrt{n}} \geq \frac{200}{3\sqrt{n}}\right) \leq 0.05,$$

or, using a normal approximation,

$$1 - \Phi\left(\frac{200}{3\sqrt{n}}\right) \leq 0.05,$$

and

$$\Phi\left(\frac{200}{3\sqrt{n}}\right) \geq 0.95.$$

From the normal tables, we obtain $\Phi(1.65) \approx 0.95$, and therefore,

$$\frac{200}{3\sqrt{n}} \geq 1.65,$$

which finally yields $n \leq 1632$.

(c) The event $N \geq 220$ (it takes at least 220 days to exceed 1000 gadgets) is the same as the event $S_{219} \leq 1000$ (no more than 1000 gadgets produced in the first 219 days). Thus,

$$\begin{aligned}
\mathbf{P}(N \geq 220) &= \mathbf{P}(S_{219} \leq 1000) \\
&= \mathbf{P}\left(\frac{S_{219} - 5 \cdot 219}{3\sqrt{219}} \leq \frac{1000 - 5 \cdot 219}{3\sqrt{219}}\right) \\
&= 1 - \Phi(2.14) \\
&= 1 - 0.9838 \\
&= 0.0162.
\end{aligned}$$