Solution to Problem 7.1. We construct a Markov chain with state space $S = \{0, 1, 2, 3\}$. We let $X_n = 0$ if an arrival occurs at time n. Also, we let $X_n = i$ if the last arrival up to time n occurred at time $n - i$, for $i = 1, 2, 3$. Given that $X_n = 0$, there is probability 0.2 that the next arrival occurs at time $n + 1$, so that $p_{00} = 0.2$, and $p_{01} = 0.8$. Given that $X_n = 1$, the last arrival occurred at time $n - 1$, and there is zero probability of an arrival at time $n+1$, so that $p_{12} = 1$. Given that $X_n = 2$, the last arrival occurred at time $n - 2$. We then have

$$\begin{aligned} p_{20} &= \mathbf{P}(X_{n+1} = 0 \,|\, X_n = 2) \\ &= \mathbf{P}(T = 3 \,|\, T \geq 3) \\ &= \frac{\mathbf{P}(T = 3)}{\mathbf{P}(T \geq 3)} \\ &= \frac{3}{8}, \end{aligned}$$

and $p_{23} = 5/8$. Finally, given that $X_n = 3$, an arrival is guaranteed at time $n + 1$, so that $p_{30} = 1$.

2. (a) Recurrent: 1, 2, 4, 5 , 6; Transient: 3; Periodic: 4,5,6.
   (b) 0.2^n
   (c) This is a geometric random variable with parameter $p = 0.5 + 0.3$. Hence, the expected number of trials up to and includ ing the trial on which the process leaves state 3 is $\mathbf{E}[X] = 1/p = 5/4$.
   (d) 3/8
   (e) $\mathbf{P}(A) = 0.3 + 0.2^3 0.3 + 0.2^6 0.3 + 0.2^9 0.3 = 0.3024$.
   (f) $0.3/\mathbf{P}(A) = 0.992$.

3. Problem 7.13, page 385 in textbook. See online solutions.