9.1. $\int_0^1 \frac{x\ln(x+\sqrt{1+x^2})}{(1+x^2)^2}\,dx = \left\{ \begin{array}{l} u=\ln(x+\sqrt{1+x^2}) \\ du = \frac{1+\frac{x}{\sqrt{1+x^2}}}{x+\sqrt{1+x^2}}\,dx \\ \quad = \frac{\frac{\cancel{\sqrt{1+x^2}+x}}{\sqrt{1+x^2}}}{\cancel{x+\sqrt{1+x^2}}}\,dx \\ \quad = \frac{dx}{\sqrt{1+x^2}} \\ dv = \frac{x}{(1+x^2)^2}dx \\ v = \int \frac{x}{(1+x^2)^2}dx = \left\{ \begin{array}{l} t=1+x^2 \\ dt = 2x\,dx \end{array} \right\} \\ \quad = \frac{1}{2}\int \frac{dt}{t^2} = -\frac{1}{2t} = -\frac{1}{2(1+x^2)} \end{array} \right\} =$

$$= -\frac{\ln(x+\sqrt{1+x^2})}{2(1+x^2)}\Big|_0^1 + \frac{1}{2}\int_0^1 \frac{dx}{(1+x^2)\sqrt{1+x^2}} =$$

$$= -\left(\frac{\ln(1+\sqrt{2})}{4} - \cancel{\frac{\ln 1}{2}}^{0}\right) + \frac{1}{2}\int_0^1 \frac{dx}{\sqrt{1+x^2}^3} = \left\{ \begin{array}{l} x = tg\,t \\ dx = \frac{dt}{\cos^2 t} \end{array} \right\}$$

Ω: $\operatorname{arctg} 0 = 0$
Γ: $\operatorname{arctg} 1 = \frac{\pi}{4}$

$$= -\frac{\ln(1+\sqrt{2})}{4} + \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{\frac{dt}{\cos^2 t}}{\sqrt{1+tg^2 t}}\,dt =$$

$$= -\frac{\ln(1+\sqrt{2})}{4} + \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{\frac{dt}{\cos^2 t}}{\sqrt{\frac{\cos^2 t+\sin^2 t}{\cos^2 t}}^3}\,dt =$$

$$= -\frac{\ln(1+\sqrt{2})}{4} + \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{\frac{dt}{\cancel{\cos^2 t}}}{\frac{1}{\cos^3 t}}\,dt = -\frac{\ln(1+\sqrt{2})}{4} + \frac{1}{2}\int_0^{\frac{\pi}{4}} \cos t\,dt$$

$$= -\frac{\ln(1+\sqrt{2})}{4} + \frac{1}{2}\sin t\Big|_0^{\frac{\pi}{4}} = -\frac{\ln(1+\sqrt{2})}{4} + \frac{1}{2}\cdot\frac{\sqrt{2}}{2} =$$

$$= \frac{\sqrt{2}}{4} - \frac{\ln(1+\sqrt{2})}{4}$$

9.2. Идеје за решавање:

$$\int_4^5 x\sqrt{x^2-4x}\,dx = \left\{\begin{array}{l} t=x^2-4x \\ dt=(2x-4)dx \\ \quad =2(x-2)dx \end{array}\right\} =$$

$$= \frac{1}{2}\int_4^5 2(x-2+2)\sqrt{x^2-4x}\,dx =$$

$$= \frac{1}{2}\left(\int_4^5 2(x-2)\sqrt{x^2-4x}\,dx + 4\int_4^5 \sqrt{x^2-4x}\,dx\right) =$$

$$= \frac{1}{2}\left(\int_0^5 \sqrt{t}\,dt + 4\int_4^5 \sqrt{x^2-4x}\,dx\right)$$

$\longrightarrow$ нпр. Остроградски

или: $$\int_4^5 x\sqrt{x^2-4x}\,dx = \int_4^5 \frac{x^3-4x^2}{\sqrt{x^2-4x}}\,dx$$

рационалишемо

решити Остроградским