

$$1. y = x \cdot e^{\frac{1}{x-2}}$$

$$y' = \frac{(x-4)(x-1)}{(x-2)^2} e^{\frac{1}{x-2}}$$

$$y'' = \frac{5x-8}{(x-2)^4} e^{\frac{1}{x-2}}$$

$$\frac{1}{0^+} = +\infty$$

$$\frac{1}{0^-} = -\infty$$

1° Зомен  $x \neq 2 \Rightarrow D_x = \mathbb{R} \setminus \{2\}$

2° Парність

$$f(-x) = -x e^{\frac{1}{-x-2}} \neq f(x) \neq -f(x) \Rightarrow \text{ниги-ниги}$$

3° Асимптоти

ВА: точка перекида  $x=2$

$$\lim_{x \rightarrow 2^+} x e^{\frac{1}{x-2}} = 2 e^{\frac{1}{2-2}} = 2^+ \cdot e^{+\infty} = 2 \cdot e^{+\infty} = +\infty$$

$$\lim_{x \rightarrow 2^-} x e^{\frac{1}{x-2}} = 2 e^{\frac{1}{2-2}} = 2 \cdot e^{-\infty} = 2 \cdot e^{-\infty} = 0$$

$x=2$  је ВА са  
гесне  
свраче

ХА:

$$\lim_{x \rightarrow +\infty} x e^{\frac{1}{x-2}} = +\infty \cdot e^0 = \infty \cdot 1 = +\infty$$

$$\lim_{x \rightarrow -\infty} x e^{\frac{1}{x-2}} = -\infty \cdot e^0 = -\infty$$

нема ХА

КА:  $y = kx + n$

$$k = \lim_{x \rightarrow \pm\infty} \frac{x \cdot e^{\frac{1}{x-2}}}{x} = \lim_{x \rightarrow \pm\infty} e^{\frac{1}{x-2}} = e^{\frac{1}{\pm\infty}} = e^0 = 1$$

$$n = \lim_{x \rightarrow \pm\infty} (x e^{\frac{1}{x-2}} - x) = \lim_{x \rightarrow \pm\infty} x (e^{\frac{1}{x-2}} - 1) = \lim_{x \rightarrow \pm\infty} \frac{e^{\frac{1}{x-2}} - 1}{\frac{1}{x}} = \lim_{x \rightarrow \pm\infty} \frac{e^{\frac{1}{x-2}} - 1}{\frac{1}{x-2}} \cdot \frac{x}{x-2} = 1 \cdot 1 = 1$$

КА је  $y = x + 1$

4° Знак и нуле

$$y = x \cdot e^{\frac{1}{x-2}} = 0 \Rightarrow x = 0 \quad (e^{\frac{1}{x-2}} > 0, \forall x \in D_x)$$

$$y > 0, x \in (0, +\infty) \setminus \{2\}$$

$$y < 0, x \in (-\infty, 0)$$

Знак фне зависи само од  $x$

|     |           |     |     |     |           |
|-----|-----------|-----|-----|-----|-----------|
| $x$ | $-\infty$ | $-$ | $0$ | $+$ | $+\infty$ |
| $y$ | $-\infty$ | $-$ | $0$ | $+$ | $+\infty$ |

5° Монотоност и ЕВ

$$y' = 0 \Rightarrow \frac{(x-4)(x-1)}{(x-2)^2} e^{\frac{1}{x-2}} = 0$$

$$\Rightarrow (x-4)(x-1) = 0$$

$$x = 4 \vee x = 1$$

$$y = 4\sqrt{e} \quad y = 1/e$$

|       |            |            |            |            |            |
|-------|------------|------------|------------|------------|------------|
| $x-4$ | $-\infty$  | $-$        | $-$        | $+$        | $+\infty$  |
| $x-1$ | $-\infty$  | $-$        | $+$        | $+$        | $+\infty$  |
| $y'$  | $+$        | $+$        | $-$        | $+$        | $+$        |
| $y$   | $\nearrow$ | $\nearrow$ | $\searrow$ | $\nearrow$ | $\nearrow$ |

$$y \uparrow, x \in (-\infty, 1) \cup (4, +\infty)$$

$$y \downarrow, x \in (1, 4) \setminus \{2\}$$

$$\max(1, 1/e)$$

$$\min(4, 4\sqrt{e})$$

6° Котв/Котк и ПТ

$$y'' = 0 \Rightarrow \frac{5x-8}{(x-2)^4} e^{\frac{1}{x-2}} = 0 \Rightarrow 5x-8 = 0$$

$$x = 8/5$$

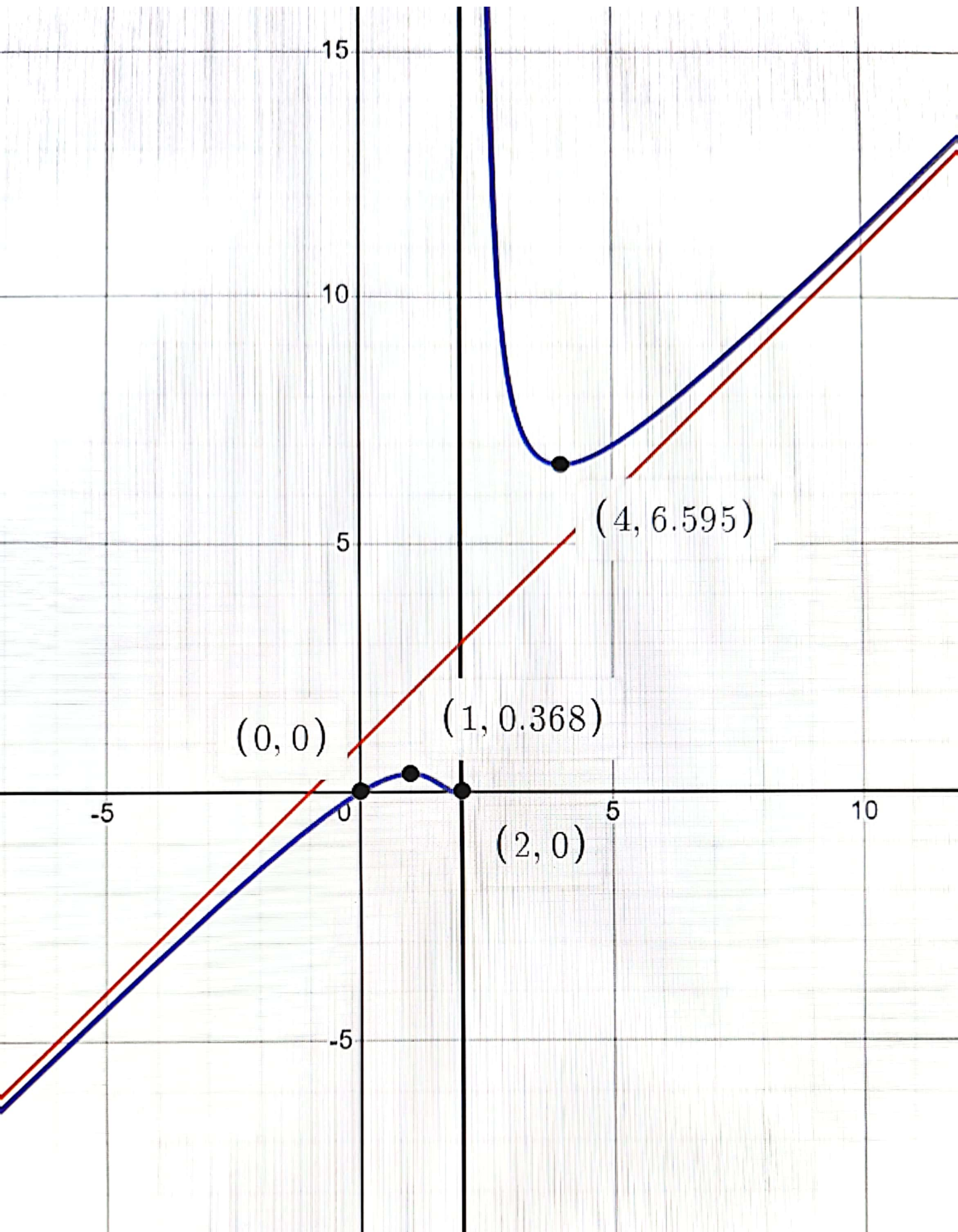
$$y = \frac{8}{5} \cdot e^{-5/2}$$

|        |           |        |        |        |           |
|--------|-----------|--------|--------|--------|-----------|
| $5x-8$ | $-\infty$ | $-$    | $0$    | $+$    | $+\infty$ |
| $y''$  | $+$       | $+$    | $-$    | $+$    | $+$       |
| $y$    | $\cap$    | $\cap$ | $\cup$ | $\cup$ | $\cup$    |

$$y \cap, x \in (-\infty, 8/5)$$

$$y \cup, x \in (8/5, +\infty) \setminus \{2\}$$

$$P(8/5, 8/5 e^{-5/2})$$



$$2. y = \frac{3x+4}{\sqrt{(x+1)(x+3)}}$$

$$y' = \frac{2x+1}{(x^2+4x+3)^{3/2}}$$

$$y'' = -\frac{y(4x+7)}{(x^2+4x+3)^{5/2}}$$

1° ДОМЕН  $(x+1)(x+3) > 0$    $\Rightarrow D_x = (-\infty, -3) \cup (-1, +\infty)$

2° Паритет  $f(-x) \neq f(x) \Rightarrow -f(x) \Rightarrow$  Ниши - Ниши

3° Асимптоти

ВА

$$\lim_{x \rightarrow -3^-} \frac{3x+4}{\sqrt{(x+1)(x+3)}} = \frac{3(-3)+4}{\sqrt{(-3+1)(-3+3)}} = \frac{-5}{\sqrt{-2 \cdot 0^-}} = \frac{-5}{0^+} = -\infty \Rightarrow x = -3 \text{ је ВА са леће стране}$$

$$\lim_{x \rightarrow -1^+} \frac{3x+4}{\sqrt{(x+1)(x+3)}} = \frac{+1}{0^+} = +\infty \Rightarrow x = -1 \text{ је ВА са десне стране}$$

ХА

$$\lim_{x \rightarrow \pm\infty} \frac{3x+4}{\sqrt{(x+1)(x+3)}} = \lim_{x \rightarrow \pm\infty} \frac{x(3+4/x)}{|x|\sqrt{(1+1/x)(1+3/x)}} \rightarrow \lim_{x \rightarrow +\infty} \frac{x(3+4/x)}{x\sqrt{(1+1/x)(1+3/x)}} = \lim_{x \rightarrow +\infty} \frac{3+4/x}{\sqrt{(1+1/x)(1+3/x)}} = 3$$

$$\lim_{x \rightarrow +\infty} \frac{x(3+4/x)}{x\sqrt{(1+1/x)(1+3/x)}} = \lim_{x \rightarrow +\infty} \frac{3+4/x}{\sqrt{(1+1/x)(1+3/x)}} = 3$$

$$\lim_{x \rightarrow -\infty} \frac{x(3+4/x)}{-x\sqrt{(1+1/x)(1+3/x)}} = \lim_{x \rightarrow -\infty} \frac{3+4/x}{-\sqrt{(1+1/x)(1+3/x)}} = -3$$

$y = 3$  је ХА за  $x \rightarrow +\infty$

$y = -3$  је ХА за  $x \rightarrow -\infty$

КА нема

4° Знак и нуле

$$y = 0 \Rightarrow 3x+4 = 0 \Rightarrow x = -\frac{4}{3} \notin D_x$$

|        |           |      |           |
|--------|-----------|------|-----------|
| $3x+4$ | -         | -    | +         |
| $y$    | -         | -    | +         |
|        | $-\infty$ | $-3$ | $+\infty$ |

$y > 0, x \in (-1, +\infty)$

$y < 0, x \in (-\infty, -3)$

Нема нуле

5° Монотоност и ЕВ

$$y' = 0 \Rightarrow 2x+1 = 0 \Rightarrow x = -\frac{1}{2}$$

$$y = \sqrt{5}$$

|        |               |                |
|--------|---------------|----------------|
| $2x+1$ | -             | <del>+</del>   |
| $y'$   | -             | <del>+</del>   |
| $y$    | $\rightarrow$ | $\nearrow$     |
|        | $-\infty$     | $-\frac{1}{2}$ |
|        |               | $+\infty$      |

$y \uparrow, x \in (-\frac{1}{2}, +\infty)$

$y \downarrow, x \in (-\infty, -3) \cup (-1, -\frac{1}{2})$

$\min(-\frac{1}{2}, \sqrt{5})$

6° Конв/конк и ПП

$$y'' = 0 \Rightarrow x(4x+7) = 0$$

$$x = 0 \vee x = -\frac{7}{4}$$

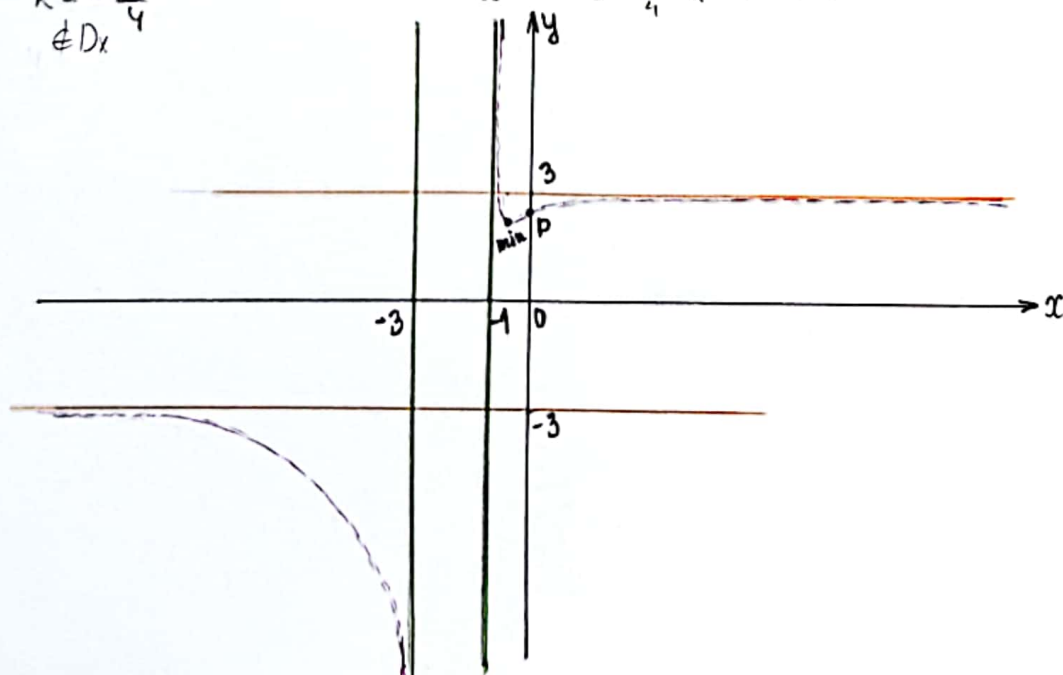
$$y = \frac{4}{\sqrt{3}}$$

|        |           |      |                |      |     |           |
|--------|-----------|------|----------------|------|-----|-----------|
| $-x$   | +         | -    | +              | -    | +   |           |
| $4x+7$ | -         | -    | +              | +    | +   |           |
| $y''$  | -         | -    | +              | +    | -   |           |
| $y$    | ∩         | ∩    | ∪              | ∪    | ∩   |           |
|        | $-\infty$ | $-3$ | $-\frac{7}{4}$ | $-1$ | $0$ | $+\infty$ |

$y \cup, x \in (-1, 0)$

$y \cap, x \in (-\infty, -3) \cup (0, +\infty)$

$P(0, \frac{4}{\sqrt{3}})$



$$3. y = \frac{1+\ln x}{1-\ln x}$$

$$y' = \frac{2}{x(1-\ln x)^2}$$

$$y'' = \frac{2(1+\ln x)}{x^2(1-\ln x)^3}$$

1° Область:  $x > 0$  и  $1 - \ln x \neq 0 \Rightarrow \ln x \neq 1 \Rightarrow x \neq e \Rightarrow D_x = (0, e) \cup (e, +\infty)$

2° Парность:  $f(-x) = \frac{1+\ln(-x)}{1-\ln(-x)}$ ,  $\ln(-x)$  не определен для  $x > 0 \rightarrow$  Нечетная-нечетная

4° Знак и нули

$$y = 0 \Rightarrow 1 + \ln x = 0 \Rightarrow x = 1/e$$

Нули  $(1/e, 0)$

|             |   |       |     |
|-------------|---|-------|-----|
| $1 - \ln x$ | + | +     | -   |
| $1 + \ln x$ | - | +     | -   |
| $y$         | - | +     | -   |
|             | 0 | $1/e$ | $e$ |

$y > 0, x \in (1/e, e)$

$y < 0, x \in (0, 1/e) \cup (e, +\infty)$

3° Асимптоты

ВА: 0 — горизонтальная и  $e$  — вертикальная

$\lim_{x \rightarrow 0^+} f(x) = \frac{-\infty}{\infty} = \lim_{x \rightarrow 0^+} \frac{1 + \frac{1}{x}}{-\frac{1}{x}} = -1 \Rightarrow x = 0$  — нуль ВА. Имеем асимптотическое приближение, приближенно  $-1$

$$\lim_{x \rightarrow e^+} f(x) = \frac{1+1^+}{1-1^+} = \frac{2}{0^-} = -\infty$$

$$\lim_{x \rightarrow e^-} f(x) = \frac{1+1^-}{1-1^-} = \frac{2}{0^+} = +\infty$$

$x = e$  — ВА с обеих сторон

ХА:

$$\lim_{x \rightarrow +\infty} \frac{1+\ln x}{1-\ln x} = \frac{+\infty}{-\infty} = \lim_{x \rightarrow +\infty} \frac{\frac{1}{x}}{-\frac{1}{x}} = -1 \Rightarrow y = -1 \text{ — ХА}$$

КА — нет

5° Монотонность и экстремумы

$$\forall x \in D_x, \frac{2}{x(1-\ln x)^2} \neq 0 \Rightarrow \text{нет экстремумов}$$

$$x > 0, (1-\ln x)^2 > 0, 2 > 0 \Rightarrow y' > 0$$

$\Rightarrow y$  — строго возрастающая

6° Конвекс/конк и ПП

$$y'' = 0 \Rightarrow 1 + \ln x = 0$$

$$\ln x = -1$$

$$x = 1/e$$

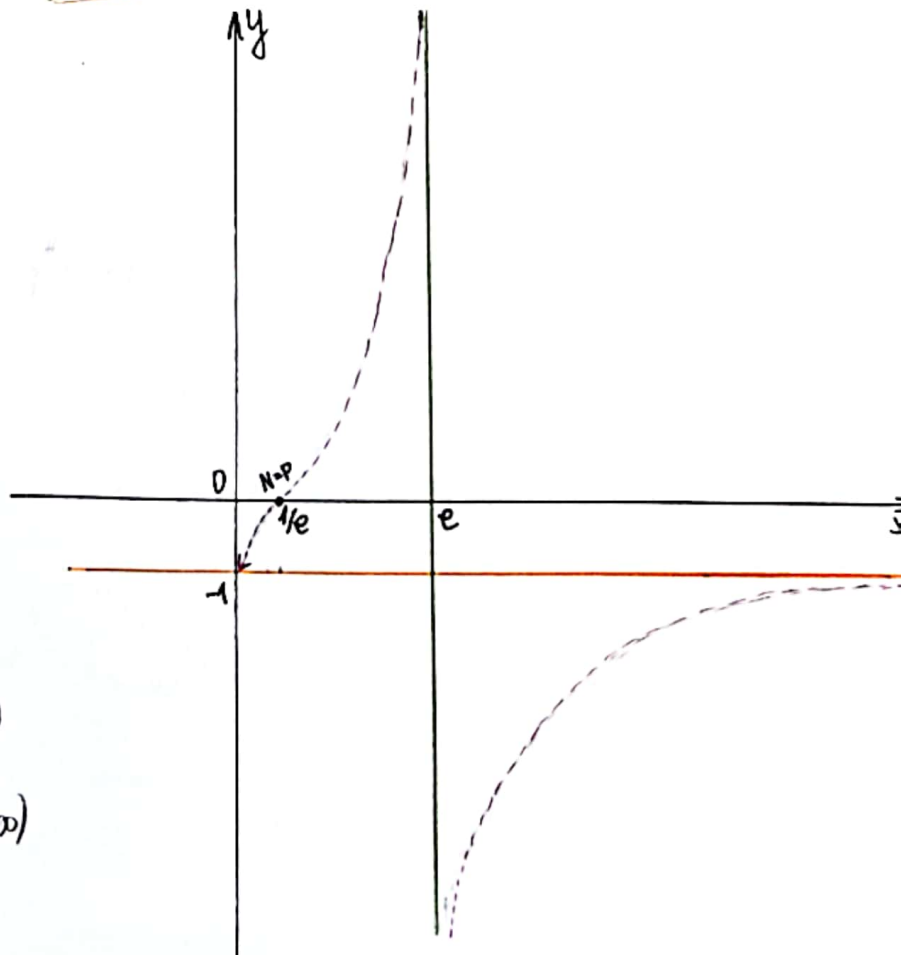
$$y = 0$$

|                 |   |       |     |
|-----------------|---|-------|-----|
| $1 + \ln x$     | - | +     | +   |
| $(1 - \ln x)^3$ | + | +     | -   |
| $y''$           | - | +     | -   |
| $y$             | ∩ | ∪     | ∩   |
|                 | 0 | $1/e$ | $e$ |

$y > 0, x \in (1/e, e)$

$y < 0, x \in (0, 1/e) \cup (e, +\infty)$

$P(1/e, 0)$



$$4. y = \arctg \frac{x-1}{x+2}$$

$$y' = \frac{3}{2x^2 + 2x + 5}$$

$$y'' = \frac{-6(2x+1)}{(2x^2 + 2x + 5)^2}$$

1° Область  $x+2 \neq 0 \Rightarrow x \neq -2 \Rightarrow D_x = \mathbb{R} \setminus \{-2\}$

2° Паритет  $f(-x) \neq f(x), f(-x) \neq -f(x) \Rightarrow$  ни чёт, ни нечёт

3° Асимптоты

ВА

$$\left. \begin{aligned} \lim_{x \rightarrow -2^+} \arctg \frac{x-1}{x+2} &= \arctg \frac{-3}{0^+} = \arctg(-\infty) = -\frac{\pi}{2} \\ \lim_{x \rightarrow -2^-} \arctg \frac{x-1}{x+2} &= \arctg \frac{-3}{0^-} = \arctg(+\infty) = \frac{\pi}{2} \end{aligned} \right\} x = -2 \text{ — гориз. ВА}$$

ХА:

$$\lim_{x \rightarrow \pm\infty} \arctg \frac{x-1}{x+2} = \arctg(1) = \frac{\pi}{4} \Rightarrow y = \frac{\pi}{4} \text{ — ХА}$$

КП нет

4° Знак и нуль

$$y=0 \Rightarrow \arctg \frac{x-1}{x+2} = 0 \Rightarrow \frac{x-1}{x+2} = 0 \Rightarrow x-1=0 \Rightarrow x=1$$

$$y > 0, x > 1 \text{ и } y < 0, x < 1 \wedge x \neq -2$$

$$\begin{array}{c|cccc} y & + & - & + & \\ \hline & -\infty & -2 & 1 & +\infty \end{array}$$

5° Монотонность и экстремумы

$$y' = \frac{3}{2x^2 + 2x + 5} \neq 0, \forall x \in D_x \Rightarrow \text{нет экстремумов}$$

$$2x^2 + 2x + 5 > 0 \quad \forall x \in \mathbb{R}$$

$$y' > 0 \Rightarrow y \nearrow \text{ на всей области}$$

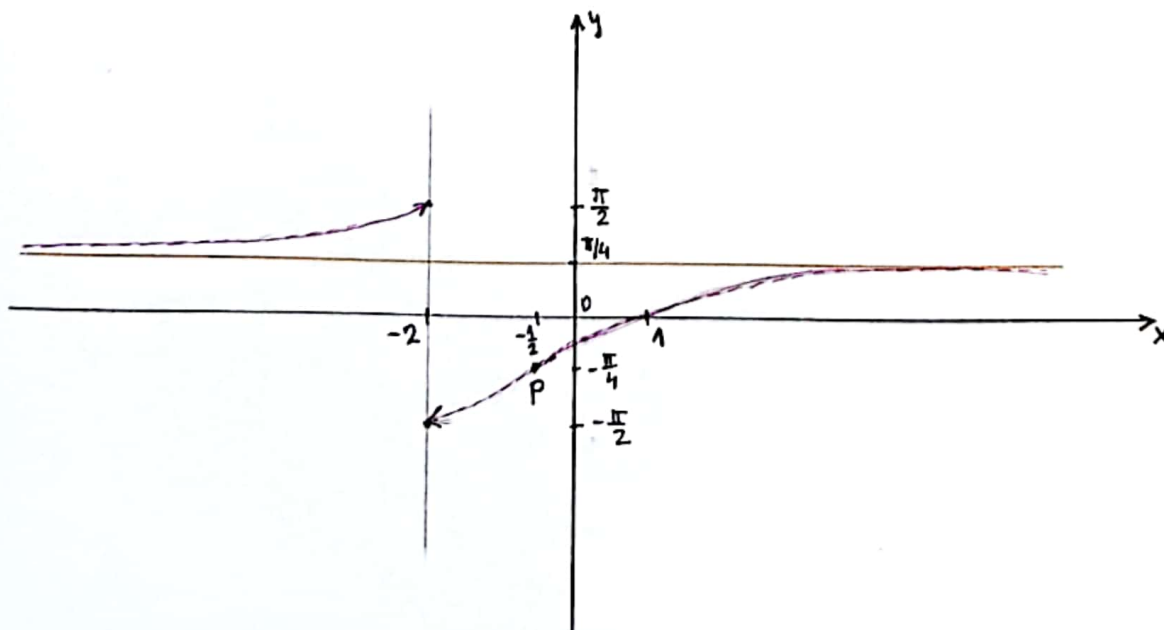
6° Конвекс/конк и ПТ

$$y'' = 0 \Rightarrow 2x+1=0 \Rightarrow x = -\frac{1}{2}$$

$$y = -\frac{\pi}{4}$$

|            |    |      |   |
|------------|----|------|---|
| $-6(2x+1)$ | +  | +    | - |
| $y''$      | +  | +    | - |
| $y$        | ∪  | ∪    | ∩ |
|            | -2 | -1/2 |   |

$y \cup, x \in (-\infty, -\frac{1}{2}) \cup (\frac{1}{2}, +\infty)$   
 $y \cap, x \in (-\frac{1}{2}, \frac{1}{2})$   
 $P(-\frac{1}{2}, -\frac{\pi}{4})$



# 5. $y = \sqrt[3]{x^3 - 3x}$

$$y' = \frac{x^2 - 1}{\sqrt[3]{x^3 - 3x}^2}$$

$$y'' = \frac{-2(x^2 + 1)}{\sqrt[3]{x^3 - 3x}^5}$$

1° Домен  $D = \mathbb{R}$

2° Парност  $f(-x) = \sqrt[3]{(-x)^3 - 3(-x)} = \sqrt[3]{-x^3 + 3x} = -\sqrt[3]{x^3 - 3x} = -f(x)$   
 ф/јо је нејарна

3° Асимптотике

ВА нема, јер нема лачака прелома

ХА

$$\lim_{x \rightarrow +\infty} \sqrt[3]{x^3 - 3x} = +\infty$$

КА.  $y = kx + n$

$$k = \lim_{x \rightarrow \pm\infty} \frac{\sqrt[3]{x^3 - 3x}}{x} = \lim_{x \rightarrow \pm\infty} \sqrt[3]{1 - \frac{3}{x^2}} = 1$$

$$\lim_{x \rightarrow -\infty} \sqrt[3]{x^3 - 3x} = -\infty$$

(нема ХА)

$$n = \lim_{x \rightarrow +\infty} (\sqrt[3]{x^3 - 3x} - x) = \lim_{x \rightarrow +\infty} x \left( \sqrt[3]{1 - \frac{3}{x^2}} - 1 \right) =$$

$$= \lim_{x \rightarrow +\infty} \frac{\sqrt[3]{1 - \frac{3}{x^2}} - 1}{\frac{1}{x}} \cdot \left( -\frac{3}{x} \right)^0 = 0$$

(коса асимптота  
 $y = x$ )

4° Знак и нуле

$$y = 0 \Rightarrow \sqrt[3]{x^3 - 3x} = 0 \Rightarrow x^3 - 3x = 0 \Rightarrow x(x^2 - 3) = 0 \Rightarrow x = 0 \vee x = \pm\sqrt{3}$$

3 нуле  $(0, 0)$ ,  $(\sqrt{3}, 0)$ ,  $(-\sqrt{3}, 0)$

$$y > 0 \Rightarrow \sqrt[3]{x^3 - 3x} > 0 \Rightarrow x^3 - 3x > 0 \Rightarrow x(x^2 - 3) > 0$$

$$y > 0 \Rightarrow x \in (-\sqrt{3}, 0) \cup (\sqrt{3}, +\infty)$$

$$y < 0 \Rightarrow \sqrt[3]{x^3 - 3x} < 0 \Rightarrow x \in (-\infty, -\sqrt{3}) \cup (0, \sqrt{3})$$

|           |           |             |     |            |           |
|-----------|-----------|-------------|-----|------------|-----------|
| $x$       | $-\infty$ | $-\sqrt{3}$ | $0$ | $\sqrt{3}$ | $+\infty$ |
| $x^2 - 3$ | +         | -           | -   | +          | +         |
| $y$       | -         | +           | -   | +          | +         |

5° Монотоност и ЕВ

$$y' = 0 \Rightarrow x^2 - 1 = 0 \Rightarrow x = \pm 1$$

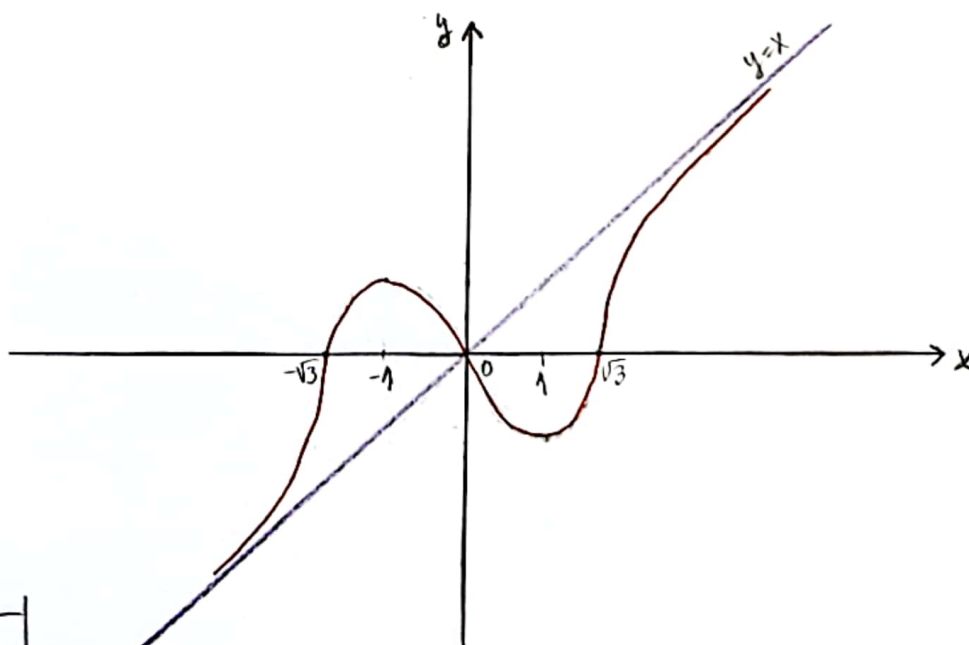
|           |   |   |   |
|-----------|---|---|---|
| $x^2 - 1$ | + | - | + |
| $y'$      | + | - | + |
| $y$       | ↗ | → | ↗ |

$$y \uparrow, x \in (-\infty, -1) \cup (1, +\infty)$$

$$y \downarrow, x \in (-1, 1)$$

$$\max(-1, -\sqrt{2})$$

$$\min(1, \sqrt{2})$$



6°  $y'' \neq 0, \forall x$

|                    |   |   |   |   |
|--------------------|---|---|---|---|
| $-2(x^2 + 1)$      | - | - | - | - |
| $(x^3 - 3x)^{5/3}$ | - | + | - | + |
| $y''$              | + | - | + | - |
| $\vartheta$        | ∪ | ∩ | ∪ | ∩ |

$$y \cup, x \in (-\infty, -\sqrt{3}) \cup (0, \sqrt{3})$$

$$y \cap, x \in (-\sqrt{3}, 0) \cup (\sqrt{3}, +\infty)$$

$$P_1(-\sqrt{3}, 0), P_2(0, 0)$$

$$P_3(\sqrt{3}, 0)$$

3 преводне лачке! нуле ф/јо