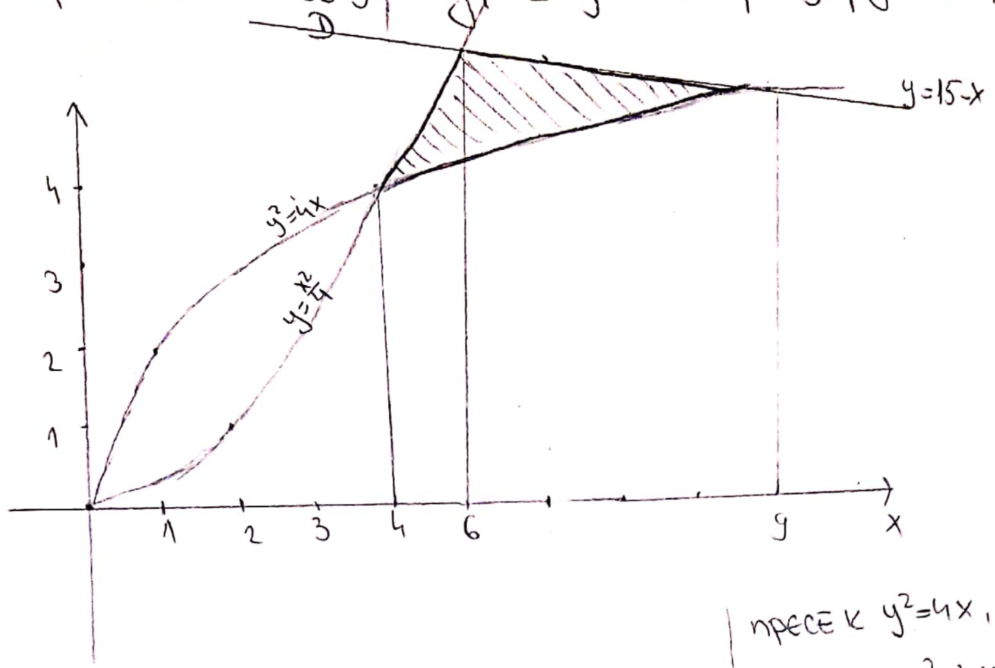
5 Израчунати $\iint_D y\,dx\,dy$, где је $D=\{(x,y) \mid y^2\ge 4x,\ 4y\le x^2,\ x+y\le 15\}$




$y^2=4x \Rightarrow$

| x | 0 | 1 | 4 |
|---|---|---|---|
| y | 0 | 2 | 4 |

$(3,1)$: $1^2\ge 4\cdot 3$, $1\ge 12$ ⊥

$4y=x^2 \Rightarrow y=\frac{x^2}{4}$

| x | 0 | 2 | 4 |
|---|---|---|---|
| y | 0 | 1 | 4 |

$(7,4) \Rightarrow 4\cdot 4\le 7^2$, $16\le 49$ T

$y=15-x$

| x | 0 | 6 | 9 | 15 |
|---|---|---|---|---|
| y | 15 | 9 | 6 | 0 |

$(4,4) \Rightarrow 4+4\le 15$, $8\le 15$ T

$y^2=4x \Rightarrow y=\pm\sqrt{4x} \Rightarrow y=2\sqrt{x}$

пресек $y^2=4x,\ 4y=x^2 \Rightarrow x=4,\ y=4$

пресек $y^2=4x,\ y=15-x$

$(15-x)^2=4x \Rightarrow 225-30x+x^2=4x$

$x^2-30x-4x+225=0$

$x^2-34x+225=0$

$x_1=9 \Rightarrow y_1=6 \Rightarrow (9,6)$

$\cancel{x_2=25}$

пресек $y=15-x,\ y=\frac{x^2}{4}$

$\frac{x^2}{4}=15-x \ /\cdot 4 \Rightarrow x^2=60-4x$

$x^2+4x-60=0$

$x_1=6 \Rightarrow y_1=9 \Rightarrow (6,9)$

$\cancel{x_2=-10}$

$$\iint_D y\,dx\,dy=\int_4^6 dx\int_{2\sqrt{x}}^{\frac{x^2}{4}} y\,dy+\int_6^9 dx\int_{2\sqrt{x}}^{15-x} y\,dy=$$

$$=\int_4^6\left(\frac{y^2}{2}\Big|_{2\sqrt{x}}^{\frac{x^2}{4}}\right)dx+\int_6^9\left(\frac{y^2}{2}\Big|_{2\sqrt{x}}^{15-x}\right)dx=\int_4^6\frac12\left(\left(\frac{x^2}{4}\right)^2-(2\sqrt{x})^2\right)dx+\int_6^9\frac12\left((15-x)^2-(2\sqrt{x})^2\right)dx=$$

$$=\frac12\left(\int_4^6\left(\frac{x^4}{16}-4|x|\right)dx+\frac12\int_6^9\left((15-x)^2-4|x|\right)dx=\frac{1}{32}\int_4^6 x^4dx-2\int_4^6 x\,dx+\frac12\int_6^9(15-x)^2dx-2\int_6^9 x\,dx=$$

$$=\left\{\begin{matrix} t=15-x \\ dt=-dx\end{matrix}\ \middle|\ \begin{matrix} x=6\Rightarrow t=9 \\ x=9\Rightarrow t=6\end{matrix}\right\}=\frac{1}{32}\frac{x^5}{5}\Big|_4^6-2\frac{x^2}{2}\Big|_4^6+\frac12\int_9^6 t^2(-dt)-2\frac{x^2}{2}\Big|_6^9=$$

$$=\frac{1}{160}(6^5-4^5)-(6^2-4^2)+\frac12\int_6^9 t^2dt-(9^2-6^2)=\frac{1}{160}(7776-1024)-(36-16)+\frac12\frac{t^3}{3}\Big|_6^9-(81-36)=$$

$$=\frac{1}{160}\cdot 6752-20+\frac16(9^3-6^3)-45=\frac{211}{5}-65+\frac16\cdot(729-216)=\frac{211}{5}-65+\frac16\cdot 513=$$

$$=\frac{211}{5}-65+\frac{171}{2}=\frac{422}{10}+\frac{855}{10}-\frac{650}{10}=\frac{627}{10}$$

6 Израчунати $\iint\limits_D e^{\frac{x}{y}}\,dx\,dy$, где је $D=\{(x,y)\mid \sqrt{x}\ge y,\ x\le 2y\}$

$\sqrt{x}=y \Rightarrow x=y^2$

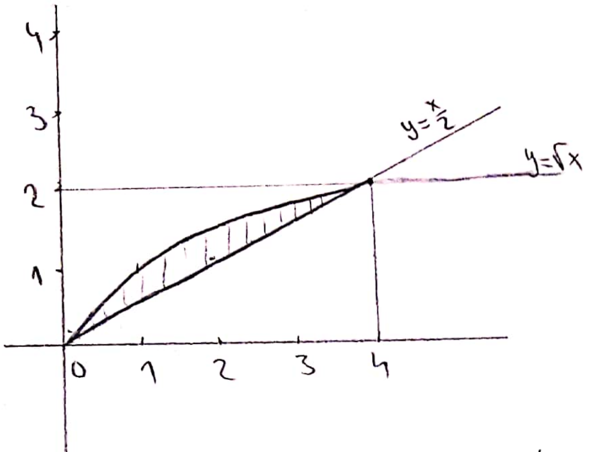
| x | 0 | 1 | 4 |
|---|---|---|---|
| y | 0 | 1 | 2 |

$(\overset{x}{3},\overset{y}{0}) \Rightarrow \sqrt{3}\ge 0$ (T)

$2y=x \Rightarrow y=\frac{x}{2}$

| x | 0 | 2 | 4 |
|---|---|---|---|
| y | 0 | 1 | 2 |

$(\overset{x}{2},\overset{y}{2}) \Rightarrow 2\le 2\cdot 2 \Rightarrow 2\le 4$ (T)




$$\iint\limits_D e^{\frac{x}{y}}dx\,dy=\int_0^2 dy\int_{y^2}^{2y} e^{\frac{x}{y}}dx=\left\{\begin{array}{l} t=\frac{x}{y} \\ dt=\frac{dx}{y}\end{array}\quad dx=y\,dt \;\middle|\; \begin{array}{l} x=y^2,\ t=\frac{y^2}{y}=y \\ x=2y,\ t=\frac{2y}{y}=2\end{array}\right\}=\int_0^2 dy\int_y^2 e^t\cdot y\,dt=$$

$$=\int_0^2 y e^t\Big|_y^2 dy=\int_0^2 y(e^2-e^y)dy=\int_0^2 e^2 y\,dy-\int_0^2 y e^y dy=\left[\begin{array}{ll} u=y & dv=e^y dy \\ du=dy & v=e^y\end{array}\right]=$$

$$=e^2\frac{y^2}{2}\Big|_0^2-\left(ye^y\Big|_0^2-\int_0^2 e^y dy\right)=\frac{e^2}{2}(2^2-0^2)-\left(2e^2-0-e^y\Big|_0^2\right)=\frac{e^2}{2}\cdot 4-\left(2e^2-(e^2-e^0)\right)=$$

$$=2e^2-(2e^2-e^2+1)=2e^2-2e^2+e^2-1=\boxed{e^2-1}$$